**2019/TDC/EVEN/MTMHC–201T/029**

**TDC (CBCS) Even Semester Exam., 2019**

# MATHEMATICS

**( 2nd Semester )**

Course No. : MTMHCC–201T

**( Real Analysis )**

*Full Marks* : 70

*Pass Marks* : 28

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## UNIT—I

**1.** Answer any *two* questions : 1×2=2

*(a)* Find the supremum of the set

$$\left\{1+\frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$$

*(b)* State completeness property of $\mathbb{R}$.

(c) Let

$$I_n = \left(0, \frac{1}{n}\right)$$

for $n \in \mathbb{N}$. Find the value of

$$\bigcap_{n=1}^{\infty} I_n$$

*Either*

2\. (a) Prove that the following statements are equivalent : 6

(i) $S$ is countable set.

(ii) There exists a surjection of $\mathbb{N}$ onto $S$.

(iii) There exists an injection of $S$ into $\mathbb{N}$.

(b) Prove that

(i) If $a \in \mathbb{R}$, $a \neq 0$, then $a^2 > 0$

(ii) $1 > 0$

(iii) If $n \in \mathbb{N}$, then $n > 0$ 6

*Or*

3\. (a) State and prove Archimedean property. Also show that if $t > 0$, there exists $n_t \in \mathbb{N}$ such that

$$0 < \frac{1}{n_t} < t$$

1+3+2=6





(b) If $x$ and $y$ are any real numbers with $x < y$, then prove that there exists a rational number $r \in Q$ such that $x < r < y$. 4

(c) If

$$S = \left\{\frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N}\right\}$$

find inf $S$ and sup $S$. 2

UNIT—II

4\. Answer any *two* questions : 1×2=2

(a) Give an example of an open set which is not an interval.

(b) Find the derived set of the set

$$A = \left\{1, \frac{1}{2}, \frac{1}{3}, \dots\right\}$$

(c) Give an example of a set which is neither open nor closed.

*Either*

5\. (a) State and prove Bolzano-Weierstrass theorem for sets. 1+5=6

(b) Define limit point of a set. Obtain the derived set of

$$\left\{\frac{1}{m}+\frac{1}{n} : m \in \mathbb{N},\ n \in \mathbb{N}\right\}$$ 1+1=2

(c) Prove that the derived set of a set is closed. 4

*Or*

6. (a) Prove that intersection of any finite number of open sets is open.
The above statement may not be true for arbitrary family of open sets. Justify with counterexample. 4+2=6

(b) Define closure of a set. Prove that the closure of a set $S$ is the intersection of all closed supersets of $S$. 1+5=6

UNIT—III

7. Answer any *two* questions : 1×2=2

(a) Give an example of a bounded sequence that is not a Cauchy sequence.

(b) Find :

$$\lim_{n\to\infty} \frac{1+2+\cdots+n}{n^2}$$

(c) Define monotone sequence.





*Either*

8. (a) Define limit of a sequence. Let $X=(x_n)$ and $Y=(y_n)$ be sequences of real numbers that converge to $x$ and $y$ respectively, then show that the sequence $X+Y$ and $X \cdot Y$ converges to $x+y$ and $x \cdot y$ respectively. 1+2+3=6

(b) State and prove monotone convergence theorem. 1+5=6

*Or*

9. (a) State and prove squeeze theorem on limits. Also prove that

$$\lim_{n\to\infty}\left(\frac{\sin n}{n}\right)=0$$ 1+3+2=6

(b) Establish the convergence or the divergence of the sequence $(y_n)$, where

$$y_n=\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}$$

for $n \in \mathbb{N}$. 4

(c) Define bounded sequence. Give an example of a bounded sequence which is not convergent. 1+1=2

## UNIT—IV

**10.** Answer any *two* questions : 1×2=2

*(a)* State Bolzano-Weierstrass theorem for sequence.

*(b)* Give an example of an unbounded sequence that has a convergent subsequence.

*(c)* Define Cauchy sequence.

*Either*

**11.** *(a)* Let $X=(x_n)$ be a sequence of real numbers. Then prove that the following statements are equivalent : 6

*(i)* The sequence $X=(x_n)$ does not converge to $x \in \mathbb{R}$.

*(ii)* There exists an $\varepsilon_0 > 0$ such that for any $k \in \mathbb{N}$, there exists $n_k \in \mathbb{N}$ such that $n_k \geq k$ and $|x_{n_k} - x| \geq \varepsilon_0$.

*(iii)* There exists an $\varepsilon_0 > 0$ and a subsequence $X' = (x_{n_k})$ of $X$ such that $|x_{n_k} - x| \geq \varepsilon_0$ for all $k \in \mathbb{N}$.

*(b)* Show that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence. 3

*(c)* Show that a Cauchy sequence of real numbers is bounded. 3





*Or*

**12.** *(a)* Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. 6

*(b)* Show that the sequence $(y_n)$, where

$$y_n = \frac{1}{\lfloor 1} - \frac{1}{\lfloor 2} + \cdots + \frac{(-1)^{n+1}}{\lfloor n}$$

is convergent. 3

*(c)* Show that a bounded, monotone increasing sequence is a Cauchy sequence. 3

## UNIT—V

**13.** Answer any *two* questions : 1×2=2

*(a)* State the necessary condition for convergence of an infinite series $\Sigma u_n$.

*(b)* If $\Sigma a_n$ with $a_n > 0$ is convergent, then is $\Sigma a_n^2$ always convergent?

*(c)* Give an example of a series which is convergent but not absolutely convergent.

*Either*

**14.** *(a)* If $\Sigma u_n$ is a positive term series such that

$$\lim_{n \to \infty} (u_n)^{\frac{1}{n}} = l$$

then prove that the series

*(i)* is convergent if $l < 1$

*(ii)* is divergent, if $l > 1$

*(iii)* the test fails to give any definite information, if $l = 1$ 6

*(b)* Test the behaviour of the following series (any *two*) : 3×2=6

*(i)* $\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \cdots, \alpha, \beta \in \mathbb{R}$

*(ii)* $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$

*(iii)* $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots$

*Or*

**15.** *(a)* State and prove Leibnitz test. 1+5=6

*(b)* Show that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$$

is conditionally convergent. 3

*(c)* Prove that every absolutely convergent series is convergent. 3

★ ★ ★

**2019/TDC/EVEN/MTMHC–202T/030**

**TDC (CBCS) Even Semester Exam., 2019**

# MATHEMATICS

**( 2nd Semester )**

Course No. : MTMHCC–202T

**( Differential Equations )**

*Full Marks* : 50
*Pass Marks* : 20

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Answer any *ten* of the following : 1×10=10

(a) Find the order and degree of the differential equation

$$\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = 2$$

(b) What is the order of the differential equation of a three-parameter family of curves?

(c) Obtain the differential equation whose solution is $y = mx + c$, where $m$ is fixed and $c$ is a parameter.

(d) Find the integrating factor of

$$x\frac{dy}{dx}+y=\sin x$$

(e) Is the differential equation

$$(x+y)^2\,dx-(y^2-2xy-x^2)\,dy=0$$

exact?

(f) Solve $xdy = ydx$.

(g) Write the differential equation for diffusion of medicine in bloodstream.

(h) Write the differential equation of simple harmonic motion.

(i) Write the necessary and sufficient condition for integrability of the total differential equation

$$Pdx+Qdy+Rdz=0$$

(j) Solve $xdy-ydx=2x^2 z\,dz$.

(k) Write Bernoulli's differential equation.

(l) Solve $(D-1)^3 y=0$.

(m) Find $\frac{1}{D^2}\cos 2x$.

(n) Find the complementary function of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=xe^x$$





Answer **five** questions, taking **one** from each Unit

## UNIT—I

2. (a) Find the differential equation of the family of circles touching the $X$-axis. 4

(b) Show that $y_1(x)=e^x\sin x$ and $y_2(x)=e^x\cos x$ are solutions of the differential equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$$

Use Wronskian to check, if the solutions are linearly independent or not. 2+2=4

3. (a) Obtain the differential equation whose solution is

$$y=a\cos x+b\sin x+\frac{1}{x}(b\cos x-a\sin x)$$ 4

(b) Show that the Wronskian of the functions $x^2$ and $x^2\log x$ are non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine the differential equation. 1+3=4

UNIT—II

4. (a) If the differential equation $Mdx + Ndy = 0$ is homogeneous of degree $n$ and $Mx + Ny \neq 0$, then show that $\frac{1}{Mx + Ny}$ will be integrating factor of the equation. 4

(b) Solve : 4

$$\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$$

5. (a) Solve : 2+2=4

(i) $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

(ii) $(x+y+1)dx + (x-y)dy = 0$

(b) Solve the differential equation by reducing it to linear form

$$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$$ 4

UNIT—III

6. Discuss the population growth model. Find the time in which (a) the population doubles and (b) the population reduces to half. 6+1+1==8

7. Discuss the simple compartmental model. 8





UNIT—IV

8. (a) Solve : 4

$$\frac{dx}{dt} + 4x + 3y = t$$
$$\frac{dy}{dt} + 2x + 5y = e^t$$

(b) Solve the total differential equation

$$yz(1+x)dx + zx(1+y) + xy(1+z)dz = 0$$ 4

9. (a) Solve : 5

$$\frac{d^2x}{dt^2} - 3x - 4y + 3 = 0$$
$$\frac{d^2y}{dt^2} + y + x + 5 = 0$$

(b) Test the integrability of the total differential equation

$$(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$$ 3

UNIT—V

10. Solve : 4+4=8

(i) $\frac{d^2y}{dx^2} - y = x\sin x$

(ii) $(x^3D^3 + x^2D^2)y = x$

**11.** *(a)* Find the particular integral of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4$$ 3

*(b)* Solve the differential equation

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$$

by the method of variation of parameters. 5

★★★