

**TDC (CBCS) Even Semester Exam., 2023**

**MATHEMATICS**

**( 6th Semester )**

**Course No. : MTMSEC-601T**

**( Analytical Geometry )**

Full Marks : 50

Pass Marks : 20

*Time : 3 hours*

*The figures in the margin indicate full marks  
for the questions*

**SECTION—A**

**Answer any *fifteen* of the following as directed :**

**1×15=15**

- 1. Find the equation of the straight line  $\frac{x}{a} + \frac{y}{b} = 2$ , when the origin is transferred to the point  $(a, b)$ .**
- 2. Write down the equation of external bisectors of the angles between the pair of straight lines of  $ax^2 + 2hxy + by^2 = 0$ .**

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3. Prove that the equation

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$$

represents a pair of perpendicular straight lines.

4. Every homogeneous equation of second degree represents \_\_\_\_.

(Fill in the blank)

5. Define radical axis.

6. What is rectangular hyperbola?

7. Write down the parametric equation of a circle.

8. Write down the position of a point (1, 2) with respect to the circle
- $x^2 + y^2 = 25$
- .

9. What do you mean by eccentricity of a conic?

10. When is a straight line said to be a normal to a conic?

11. What is the condition of tangency of a straight line
- $y = mx + c$
- to a parabola
- $y^2 = 4ax$
- ?

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( Continued )

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12. Write down the equation of a normal to the conic
- $\frac{l}{r} = 1 + e \cos \theta$
- .

13. When is a circle said to be a great circle?

14. Define skew line.

15. What do you mean by shortest distance between two lines?

16. Write down the equation of a sphere when
- $(x_1, y_1, z_1)$
- and
- $(x_2, y_2, z_2)$
- are the extremities of its diameter.

17. What do you mean by guiding curve of a cylinder?

18. Define cone.

19. Write down the general equation of second degree in
- $x, y, z$
- .

20. Define the term right circular cylinder.

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( Turn Over )

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## SECTION—B

Answer any five of the following questions :  $2 \times 5 = 10$ 

21. If the pair of straight lines  $x^2 - 2pxy - y^2 = 0$  and  $x^2 - 2qxy - y^2 = 0$  be such that each pair bisects the angles between the other pair, prove that  $pq = -1$ .

22. Transform to axes inclined at  $30^\circ$  to the original axes the equation

$$x^2 + 2\sqrt{3}xy - y^2 = 8$$

23. Find the equation of the sphere which passes through the origin and touches the sphere  $x^2 + y^2 + z^2 = 56$  at the point  $(2, -4, 6)$ .

24. Find the condition that the straight lines  $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ ,  $\frac{x}{\alpha\alpha} = \frac{y}{\alpha\beta} = \frac{z}{\alpha\gamma}$  and  $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$  will lie on a plane.

25. Find the nature of the conic  $\frac{8}{r} = 4 - 5\cos\theta$ .

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26. If the normal at the point  $(at_1^2, 2at_1)$  on the parabola  $y^2 = 4ax$  meets it again at the point  $(at^2, 2at)$ , show that  $t = -t_1 - \frac{2}{t_1}$ .

27. Write down the equation of the plane containing the first line and the line of shortest distance between the lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and}$$

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

28. Find the equation of the circle which passes through the points  $(3, 4)$  and  $(3, -6)$  and which has its centre on the straight line  $2x + 3y = 3$ .

29. Find the equation of a right circular cylinder of radius 3 and whose axis is

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$$

30. Write down the condition for general equation of second degree to represent cone.

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## SECTION—C

Answer any five of the following questions :  $5 \times 5 = 25$ 

31. Show that the area of the triangle formed by the straight lines  $ax^2 + 2hxy + by^2 = 0$  and  $lx + my = 1$  is

$$\frac{\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}$$

Find the length of the side which lies on the straight line  $lx + my = 1$ .  $3+2=5$

32. The expression  $ax^2 + 2hxy + by^2$  changes to  $a'x'^2 + 2h'x'y' + b'y'^2$  by rotation of rectangular axes about origin, then prove that  $a+b = a'+b'$  and  $ab - h^2 = a'b' - h'^2$ .

33. (a) Find the radical axis of the two circles  $x^2 + y^2 + 4x - 2y + 9 = 0$  and  $x^2 + y^2 + 2x + 3y - 5 = 0$ . 2

- (b) Prove that the circle  $x^2 + y^2 + 2ax + c^2 = 0$  and  $x^2 + y^2 + 2by + c^2 = 0$  will touch each other if  $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ . 3

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34. (a) Prove that the center of the three circles  $x^2 + y^2 = 1$ ,  $x^2 + y^2 + 6x - 2y = 1$  and  $x^2 + y^2 - 12x + 4y = 1$  lie on a straight line. 3

- (b) Find the equation of the circles which cuts orthogonally each of the three circles  $x^2 + y^2 + 2x + 17y + 4 = 0$ ,  $x^2 + y^2 + 7x + 6y + 11 = 0$  and  $x^2 + y^2 - x + 22y + 3 = 0$ . 2

35. Find the polar equation of a conic referred to a focus as pole. When will this conic be a parabola?  $4+1=5$

36. (a) Show that the straight line  $x \cos \alpha + y \sin \alpha = p$  touches the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  if  $a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$ . 3

- (b) Show that three normals can be drawn from a given point to a parabola. 2

37. Prove that the circles  $2(x^2 + y^2 + z^2) + 8x - 13y + 17z - 17 = 0$ ,  $2x + y - 3z + 1 = 0$  and  $x^2 + y^2 + z^2 + 3x - 4y + 3z = 0$ ,  $x - y + 2z - 4 = 0$  lie on the same sphere. Find its equation.  $3+2=5$

38. Find the shortest distance between the straight lines

$$\frac{x-3}{-3} = \frac{y-8}{1} = \frac{z-3}{-1} \text{ and } \frac{x+3}{3} = \frac{y+7}{-2} = \frac{z-6}{-4}$$

and the equation of the line of shortest distance. 2+3=5

39. (a) Find the equation of the cylinder generated by straight lines parallel to

$$\frac{x}{1} = \frac{y}{5} = \frac{z}{-2} \text{ the guiding curve being the}$$

conic  $x=0, y^2=6z$ . 3

- (b) Find the equation of the cone whose vertex is the origin and base is the circles  $x=a, y^2+z^2=b^2$ . 2

40. (a) A variable plane is parallel to the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0 \text{ and meets the axes in } A, B,$$

$C$  respectively. Prove that the circle  $ABC$  lies on the cone

$$\left(\frac{b}{c} + \frac{c}{b}\right)yz + \left(\frac{c}{a} + \frac{a}{c}\right)zx + \left(\frac{a}{b} + \frac{b}{a}\right)xy = 0 \quad 3$$

- (b) Find the equation of the right circular cylinder of radius 3 and whose axis is

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6} \quad 2$$

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