2023/TDC (CBCS)/EVEN/SEM/
MTMDSE–602T(A/B)/039

TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(6th Semester)

Course No. : MTMDSE–602T

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* from Option—A *or* Option—B

OPTION—A

Course No. : MTMDSE–602T (A)

(Hydrodynamics)

SECTION—A

Answer any *twenty* of the following as directed : 1×20=20

1. Define ideal fluid.

2. What do you mean by turbulent flow?

3. Define steady flow.

4. Define velocity potential.

5. What do you mean by vorticity vector?

6. What does equation of continuity signify?

7. Write the equation of continuity in Lagrangian form.

8. What is the equation of continuity of any fluid in Cartesian coordinates?

9. Write the equation of continuity in spherical polar coordinates.

10. Equation of continuity in cylindrical coordinates is _____.

(Fill in the blank)

11. Acceleration of a fluid particle is given by $\frac{\partial \vec{q}}{\partial t}$.

Is the statement true? If not, write the correct one.

12. Write the components of acceleration of a fluid particle in cylindrical coordinates.

13. What do you mean by material derivative?

14. Stream function is also known as _____ function.

(Fill in the blank)

15. Stream function exists in all types of two-dimensional motion, whether _____ or _____.

(Fill in the blanks)

16. What does equation of motion signify?

17. Write Euler's equation of motion in Y-direction.

18. If $\vec{F} = X\hat{i} + Y\hat{j} + Z\hat{k}$ and $\vec{F} = -\vec{\nabla}V$, then V is said to be _____.

(Fill in the blank)

19. Write the energy equation for incompressible fluid.

20. Write Lamb's hydrodynamical equation.

21. _____ equation is obtained by integrating Euler's equation of motion.

(Fill in the blank)

22. Write Bernoulli's equation for unsteady, irrotational motion.

23. According to Euler's momentum theorem, net rate of gain of momentum is _____.

(Fill in the blank)

24. What is D'Alembert's paradox?

25. Given

$$\int \frac{dp}{\rho} + \frac{1}{2}q^2 + V = C$$

Is the motion steady?

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Write the difference between streamlines and path lines.

27. Give examples of rotational and irrotational flows.

28. The equation of continuity in vector form is

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{q}) = 0$$

express it in the form

$$\frac{D}{Dt}(\log \rho) + \vec{\nabla} \cdot \vec{q} = 0$$

29. Obtain the equation of continuity in Lagrangian form.

30. Given the velocity field

$$\vec{q} = Ax^2 y\hat{i} + By^2 zt\hat{j} + Czt^2\hat{k}$$

Determine the component of acceleration in X-direction.

31. The velocity components of a flow is given by $u = -C$, $v = 0$. Determine the stream function.

32. State the law of conservation of momentum for a fluid motion and write Euler's equation of motion in Cartesian form.

33. Write the statement of energy equation.

34. State Bernoulli's theorem.

35. Explain Euler's momentum theorem.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. What are the methods of describing fluid motion? Explain each of them. 2+6=8

37. The velocity components for a two-dimensional fluid system can be given in the Eulerian system by

$$u = 2x + 2y + 3t, \; v = x + y + \frac{1}{2}t$$

Find the displacement of a fluid particle in the Lagrangian system.

38. Derive the equation of continuity of an incompressible fluid in the form $\vec{\nabla} \cdot \vec{q} = 0$, $\vec{q}$ being the fluid velocity.

39. *(a)* If the velocity of an incompressible fluid at the point (x, y, z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right), \; r^2 = x^2 + y^2 + z^2$$

prove that fluid motion is possible. 4

(b) A mass of fluid moves in such a way that each particle describes a circle in

one plane about a fixed axis. Show that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \theta}(\rho\omega) = 0$$

where ω is the angular velocity of a particle whose azimuthal angle is θ. 4

40. Write the relation among material, local and convective derivative and explain each term. Show also that the acceleration of a fluid particle is the material derivative of its velocity. 2+6=8

41. *(a)* In spherical coordinates (r, θ, ϕ), the velocity components are

$$V_r = \left(\frac{r^2}{t^2}\sin\phi\right), \; V_\theta = \left(\frac{r}{t}\right)\cot\theta\,\mathrm{cosec}\,\phi, \; V_\phi = \left(\frac{r}{t}\right)\sin\theta\cos\phi$$

Determine the components of acceleration of a fluid particle. 5

(b) A velocity field is given by

$$\vec{q} = -x\hat{i} + (y + t)\hat{j}$$

Find the stream function of the flow. 3

42. Deduce the equation of motion in the form

$$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p$$

43. Obtain the equation of motion in the form

$$\frac{\partial \vec{q}}{\partial t} + \vec{w} \times \vec{q} = \vec{F} - \frac{1}{\rho}\vec{\nabla} p - \frac{1}{2}\vec{\nabla} q^2$$

where $\vec{w} = \text{curl}\vec{q}$.

44. When velocity potential exists and forces are conservative and derivable from a potential Ω, the equation of motion can always be integrated and the solution is

$$\int \frac{dp}{\rho} - \frac{\partial \phi}{\partial t} + \frac{1}{2}q^2 + \Omega = F(t)$$

Prove this.

45. A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A, is delivered at atmospheric pressure at a place where the sectional area is B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth

$$\frac{S^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$$

below the pipe, S being the delivery per second.

OPTION—B

Course No. : MTMDSE–602T (B)

(Theory of Equations)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. State fundamental theory of algebra.

2. State remainder theorem.

3. What will be the nature of the roots if the signs of the terms of an equation are all positive?

4. Find the quotient and remainder when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ is divided by $x - 3$.

5. Write Descarte's rule of signs for positive roots.

6. If α and β are the roots of the equation $x^2 - 2x + 3 = 0$, then find the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

7. If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then find $\Sigma\alpha^2$.

8. Find the equation whose roots are the reciprocals of the roots of $x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$.

9. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then find the value of k.

10. Find the equation whose roots are the roots of $4x^5 - 2x^3 + 7x - 3 = 0$, when each root is increased by 2.

11. Write down the standard form of a biquadratic equation.

12. Under what transformation the equation $ax^3 + 3bx^2 + 3cx + d = 0$ reduces to $z^3 + 3Hz + G = 0$?

13. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find $\sum \frac{1}{\alpha}$.

14. If α, β, γ and δ be the roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find $\Sigma\alpha\beta$.

15. State Newton's theorem on the sums of powers of roots.

16. Define superior limit of roots.

17. Find a superior limit of the positive roots of the equation $x^4 - 5x^3 + 40x^2 - 8x + 23 = 0$.

18. If all the roots of $f(x) = ax^3 + bx^2 + cx + d$ are real, then find the number of real roots of $f'(x) = 0$.

19. State Fourier and Budan theorem.

20. Let $f(x) = x^3 - 2x - 5$. Find the first derived function $f_1(x)$.

21. State Sturm's theorem.

22. Find the condition that the roots of the equation $ax^2 + 2bx + c = 0$ are real and unequal.

23. Write the name of a method which can be used to find both the commensurable and incommensurable roots of a numerical equation.

24. Write the commensurable roots of an equation $x^4 - 4x^3 + 8x + 4 = 0$, if exists.

25. What do you mean by numerical equation?

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Find the equation whose roots are –3, –1, 4, 5.

27. Given that the equation $x^4 - 6x^3 + 8x^2 - 17x + 10 = 0$ has a root 5, find the equation containing the remaining roots.

28. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then find (p, q).

29. If the difference of the roots of $x^2 - px + 8 = 0$ be 2, then find the value of p.

30. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find $(\alpha + \beta) \cdot (\beta + \gamma) \cdot (\gamma + \alpha)$.

31. Transform the cubic $ax^3 + 3bx^2 + 3cx + d$ into the reciprocal form.

32. Find a superior limit of the positive roots of the equation

$$x^5 + 3x^4 + x^3 - 8x^2 - 51x + 18 = 0$$

33. Find the number and position of the real roots of the equation

$$x^4 - 2x^3 - 7x^2 + 10x + 10 = 0$$

34. Find all the commensurable roots of $2x^3 - 31x^2 + 112x + 64 = 0$.

35. Find all the roots of the equation $x^4 - 2x^3 - 19x^2 + 68x - 60 = 0$ which lie between –6 and 6.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. *(a)* Express $3x^3 - 4x^2 + 5x + 6$ as a polynomial in $x + 1$. 4

(b) Show that the equation $x^3 + x^2 - 5x - 1 = 0$ has one positive root in (1, 2) and two negative roots in (–1, 0) and (–3, –1). 4

37. *(a)* Apply Descarte's rule of signs to find the nature of the roots of the equation $x^4 + qx^2 + rx - s = 0$, where q, r, s are positive. 4

(b) Show that the equation $x^3 - qx + r = 0$, where q, r are positive, has one negative root, and the other two roots are either imaginary or both positive. 4

38. *(a)* If the roots of the equation

$$x^3 - px^2 + qx - r = 0$$

be in harmonic progression, show that the mean root is $\frac{3r}{q}$. 4

(b) Find the equation whose roots are the cubes of the roots of the equation $x^4 - 2x^3 + x^2 + 3x - 1 = 0$. 4

39. *(a)* Solve the equation

$$6x^3 - 11x^2 + 6x - 1 = 0$$

whose roots are in harmonic progression. 4

(b) Let α, β, γ be the roots of the equation $x^3 + 2x^2 + 1 = 0$. Find the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$. 4

40. *(a)* Solve $x^3 - 18x - 35 = 0$ by Cardan's method. 4

(b) If α, β, γ be the roots of the equation $x^3 + px + q = 0$, then find $\sum \frac{1}{\alpha + \beta}$. 4

41. Derive the complete algebraic solution of the cubic equation

$$ax^3 + 3bx^2 + 3cx + d = 0$$

42. *(a)* Apply Sturm's theorem to analyze the equation $x^4 - 4x^3 + 7x^2 - 6x - 4 = 0$. 4

(b) Calculate Sturm's functions for the equation $3x^5 + 5x^3 + 2 = 0$ and show that four roots are imaginary. 4

43. Let

$$x^n + p_1 x^{n-1} + p_2 x^{n-2} + \cdots + p_{n-1} x + p_n = 0$$

be any equation. If the first negative term be $-p_r x^{n-r}$ and the greatest negative coefficient be $-p_k$, show that $\sqrt[r]{p_k} + 1$ is a superior limit of the positive roots.

44. *(a)* Calculate Sturm's remainders for biquadratic equation

$$z^4 + 6Hz^2 + 4Gz + a^2 I - 3H^2 = 0$$ 4

(b) Apply Budan's method to separate the roots of the equation

$$x^4 - 16x^3 + 69x^2 - 70x - 42 = 0$$ 4

45. *(a)* Find the positive root of the equation $x^3 - 2x - 5 = 0$ by using continued fraction method. 4

(b) Prove that the roots of the equation $x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$ are all real, and solve it when two of the quantities a, b, c become equal. 4

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