2023/TDC(CBCS)/EVEN/SEM/
MTMDSC/GE-201T/030

TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

(2nd Semester)

Course No. : MTMDSC/GE-201T

(Differential Equations)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *twenty* questions : 1×20=20

1. Is the differential equation
$$(ax + hy + g)dx + (hx + by + f)dy = 0$$
exact?

2. Find the value of $d\left(\frac{y^2}{x}\right)$.

3. Define integrating factor.

4. If

$$\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)=f(y)$$

what is the integrating factor of $Mdx+Ndy=0$?

5. Write the equation

$$\left(\frac{dy}{dx}\right)^2(x^2-a^2)-2\left(\frac{dy}{dx}\right)xy+y^2-b^2=0$$

in Clairaut's form.

6. Give an example of 2nd-order linear differential equation with constant coefficient.

7. Define linearly independent solution.

8. Under what condition, $y=e^x$ is the solution of

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0\ ?$$

9. Verify that e^{2x} is one of the solutions of

$$\frac{d^3y}{dx^3}-8y=0$$

10. Find the value of $w(x, 2x^2)$.

11. Solve :

$$\frac{d^2y}{dx^2}-4y=0$$

12. Solve :

$$\frac{d^2y}{dx^2}=e^{ax}$$

13. Find the CF of the differential equation $(D^2+6D+5)y=16e^{3x}$, where $D=\frac{d}{dx}$.

14. Find the PI of the differential equation $(D^2-1)y=\sin 2x$, where $D=\frac{d}{dx}$.

15. If $f(D^2)=\cos ax$, write PI, provided $f(-a^2)\neq 0$.

16. Define total differential equation.

17. Is the equation $ydx+xdy+zdz=0$ integrable?

18. Solve :

$$\frac{dx}{z}=\frac{dy}{0}=\frac{dz}{-x}$$

19. Give an example of a Cauchy-Euler equation.

20. Define ordinary simultaneous differential equations.

21. Define degree of a partial differential equation.

22. Write the degree of the partial differential equation

$$\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\frac{\partial z}{\partial x}$$

23. Write the order of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$$

24. Define linear partial differential equation.

25. Whether the partial differential equation $z\left(\frac{\partial z}{\partial x}\right) + \frac{\partial z}{\partial y} = x$ is linear or non-linear?

SECTION—B

Answer any *five* questions : 2×5=10

26. Solve :

$$(e^y + 1)\cos x dx + e^y \sin x dy = 0$$

27. Find integrating factor of the differential equation

$$(x^2 + y^2 + 1)dx - 2xydy = 0$$

28. Show that $\sin x$ and $\sin x - \cos x$ are linearly independent solution of

$$\frac{d^2y}{dx^2} + y = 0$$

29. Show that the Wronskian of $e^{ax}\cos bx$ and $e^{ax}\sin bx, (b \neq 0)$ is be^{2ax}.

30. Solve $(D^2 + 4)y = x^2$, where $D = \frac{d}{dx}$.

31. Solve $(D^2 - 4D + 4)y = x^3 e^{2x}$, where $D = \frac{d}{dx}$.

32. Show that

$$(2x + y^2 + 2xz)dx + 2xydy + x^2 dz = 0$$

is integrable.

33. Solve :

$$\frac{xdx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}$$

34. Eliminate the arbitrary constants a and b from $z = (x-a)^2 + (y-b)^2$ to form the partial differential equation.

35. Form a partial differential equation by eliminating arbitrary constants h and k from $z = hx + ky + hk$.

SECTION—C

Answer any *five* questions : 8×5=40

36. *(a)* State and prove the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact. 5

(b) Solve $P(P^2 + xy) = P^2(x + y)$, where $P = \frac{dy}{dx}$. 3

37. *(a)* Solve $(x^3 - 2y^2)dx + 2xydy = 0$. 3

(b) Solve $y = px + \sqrt{a^2p^2 + b^2}$, where $p = \frac{dy}{dx}$. 5

38. *(a)* Determine the differential equation with the linearly independent solution x, x^2 and x^3. 3

(b) Show that the Wronskian of the functions x^2, $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine this equation. 5

39. *(a)* Verify that the functions $\sin x$, $\cos x$ and $\sin 2x$ are linearly independent. 4

(b) Prove that the Wronskian of the functions $e^{m_1x}, e^{m_2x}, e^{m_3x}$ is equal to $(m_1 - m_2)(m_2 - m_3)(m_3 - m_1)e^{(m_1+m_2+m_3)x}$. 4

40. *(a)* Solve $(x^2D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$, where $D = \frac{d}{dx}$. 4

(b) Solve $(D^2 + a^2)^2 y = \sin ax$, where $D = \frac{d}{dx}$. 4

41. *(a)* Solve $(D^2 - 6D + 9)y = 6e^{3x}$, where $D = \frac{d}{dx}$. 4

(b) Solve : 4

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$$

42. *(a)* Solve : 4

$$x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} = x$$

(b) Solve : 4

$$\frac{dx}{x^2+2y^2}=\frac{dy}{-xy}=\frac{dz}{xz}$$

43. (a) Prove that the necessary condition for integrability of total differential equation $Pdx+Qdy+Rdz=0$ is

$$P\left(\frac{\partial Q}{\partial z}-\frac{\partial R}{\partial y}\right)+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=0$$ 5

(b) Solve : 3

$$\frac{dx}{dt}=2y+x,\ \frac{dy}{dt}=y$$

44. (a) Find the partial differential equation of the set of all right circular cones whose axes coincide with z-axis. 4

(b) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z,\ x^2+y^2-z^2)=0$. 4

45. (a) Form a partial differential equation by eliminating arbitrary functions f and F from $y=f(x-at)+F(x+at)$. 4

(b) Find the partial differential equation of all planes which are at a constant distance a from the origin. 4

★★★