

2023/TDC(CBCS)/EVEN/SEM/  
MTMHCC-202T/029

TDC (CBCS) Even Semester Exam., 2023

MATHEMATICS

( Honours )

( 2nd Semester )

Course No. : MTMHCC-202T

( Differential Equations )

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

**SECTION—A**

Answer any ten of the following questions :  $2 \times 10 = 20$

1. Write the degree and order of the following differential equations :

(i)  $k \frac{d^2 y}{dx^2} = \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}$

(ii)  $dy = (x + \sin x) dx$

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2. Obtain a differential equation satisfied by the family of concentric circles.
3. Prove that  $\sin 2x$  and  $\cos 2x$  are solutions of  $y'' + 4y = 0$  and these solutions are linearly independent.
4. Determine the necessary condition for a differential equation of first order and first degree to be exact.
5. Find the integrating factor of  
 $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$
6. Solve :  
 $x^2 \left( \frac{dy}{dx} \right) + xy = \sqrt{1 - x^2y^2}$
7. What do you mean by simple compartmental model?
8. Formulate the differential equation for exponential decay model.
9. Write two assumptions for limited growth with harvesting model.
10. Solve :

$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$$

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11. Show that

$$(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0$$

is integrable.

12. Solve :

$$zydx = zxdy + y^2dz$$

13. Find the PI of  $(D^2 - 3D + 2)y = \cosh x$ ;  $D \equiv \frac{d}{dx}$ .

14. Solve :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$$

15. Find PI of  $(D^2 + a^2)y = \sin ax$ .

## SECTION—B

Answer any *five* of the following questions :  $6 \times 5 = 30$

16. (a) Find the differential equation of all circles of radius  $a$

$$(x - h)^2 + (y - k)^2 = a^2 \quad 3$$

- (b) Prove that the functions  $1, x, x^2$  are linearly independent. Hence form the differential equation whose solutions are  $1, x, x^2$ . 3

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17. (a) Form the differential equation of the curves  $xy = ae^x + be^{-x} + x^2$ . 3

(b) Show that  $y = 2\sin x + 3\cos x$  is an explicit solution of the differential equation  $\frac{d^2y}{dx^2} + y = 0$ , for all real  $x$ . 3

18. Define integrating factor. Find the integrating factor of the differential equation  $(xy \sin xy + \cos xy) y dx + (xy \sin xy - \cos xy) x dy = 0$  and hence solve it. 1+1+4=6

19. (a) Solve : 3  
 $(1-x^2)\left(\frac{dy}{dx}\right) + 2xy = x\sqrt{1-x^2}$

(b) Solve : 3  
 $x\left(\frac{dy}{dx}\right) + y \log y = xye^x$

20. Discuss the simple compartmental model. 6

21. Let in a lake the pollution level is 7%. If the concentration of the incoming water is 2% and 10000 litres of water per day is allowed to enter the lake, find the time when pollution level is 5% and volume of the lake is 200000 litres. Also, find pollution level after 32 days. 6

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22. (a) Solve : 3

$$x dx + y dy - \sqrt{(a^2 - x^2 - y^2)} dz = 0$$

(b) Solve : 3

$$\frac{dx}{dt} - 7x + y = 0, \quad \frac{dy}{dt} - 2x - 5y = 0$$

23. (a) Solve : 3

$$yz \log z dx - zx \log z dy + xy dz = 0$$

(b) Solve  $\frac{dx}{dt} = -\omega y$  and  $\frac{dy}{dt} = \omega x$ . Also, show that the point  $(x, y)$  lies on a circle. 3

24. (a) Solve : 3

$$\frac{d^3y}{dx^3} - y = (e^x + 1)^2$$

(b) Solve the equation  $\frac{d^2y}{dx^2} = a + bx + cx^2$ , given that  $\frac{dy}{dx} = 0$  when  $x = 0$  and  $y = d$  when  $x = 0$ . 3

25. (a) Solve :

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$$\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12 \log x}{x^2}$$

(b) Use the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = \frac{e^x}{1+e^x} \quad 4$$

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