A STUDY ON CONJUGATE GRADIENT METHODS AND THEIR MODIFICATIONS

A Thesis submitted to Assam University, Silchar in partial fulfilment of the requirement for the Degree of Doctor of Philosophy in Mathematics

By

BIPRAJIT NATH

Ph.D Registration Number

Ph.D/1881/2012 Date-09/04/2012

DEPARTMENT OF MATHEMATICS

ELBERT EINSTEIN SCHOOL OF PHYSICAL SCIENCES

ASSAM UNIVERSITY,SILCHAR

Silchar-788011, India

Year of Submission-2016

Chapter 7

Results and discussion

The followings are the results and conclusion for the work:

1. From the investigation and discussion in chapter 4 it is clear that the convergent solution can be obtained for unconstrained non-linear programming problems using non-linear Conjugate Gradient methods by taking an initial search direction other than the direction $d_0 = -g_0$. In the present thesis we have taken the initial search direction slightly deflect from $d_0 = -g_0$ and with this deflected direction also we have succeeded to achieve a convergent solution. The future prospect of the present work is open as the nature of γ needs more investigation.

2. In chapter 5 a new β has been proposed and the efficiency of this new proposed β is analysed in light of the experimental values.

With the new value of β in (5.10), the experiment has been carried out in MATLAB and the optimum value is correct upto three decimal places. NI indicates the number of iterations. In the experiments the value of γ is taken in the interval (0,1) and close to 1, the value of t is taken from the interval [1,2].

Experimental results for various Test Functions are as follows:

Problem	Dim β	Proposed (NI)	β^{FR} (NI)	β^{HS} (NI)
Rosenbrock	100/200/300/400/500	10/12/13/15/19	15/17/19/22/23	18/20/23/26/28
Sphere	100/200/300/400/500	11/12/14/18/20	13/16/17/18/19	14/16/18/19/23
Hyper Ellipsoid	100/200/300/400/500	19/22/24/28/31	20/25/29/33/39	20/24/27/20/35

Zakharov	100/200/300/400/ 500	20/23/27/29/34	21/26/28/32/ 37	19/25/29/ 36/41
Trigonometric	100/200/300/400/ 500	20/23/27/29/34	21/26/28/32/ 37	19/25/29/ 36/41
Extended Powell	100/200/300/400/ 500	54/63/71/88/97	61/77/86/95/ 108	69/82/98/ 104/112
Broyden tridiagonal	100/200/300/400/ 500	37/36/34/31/30	41/39/38/36/ 35	44/41/38/ 37/35
Broyden banded	100/200/300/400/ 500	21/22/26/27/28	/39/38/36/35/ 33	39/38/37/ 35/32

From the above table it is clear that, with the new expression of β given in (5.10), the efficiency of the CG method improved in comparison with the other values of β.

3. From the above investigation in Chapter-6, it is clear that if we take the value of the difference between the consecutive pair of step lengths from the sequence $\{\alpha_k\}$, we observe that the value of each pair is different from the value of the next consecutive pair for different values of $x_0, x_1, x_2, ...$ and $g_0, g_1, g_2, ...$ i.e. the sequence of step length tends to vary in a totally unpredictable manner.

Future Directions: Conjugate Gradient method have been experienced an intensive theoretical and computational research for last few decades, there is a huge research scope in this area. There are some very important open problems, which requires further investigations.Out of these we have investigated three open problems and found some new and better result.

From the investigation and discussion in Chapter 4 it is clear that the convergent solution can be obtained for unconstrained non-linear programming problems using non-linear Conjugate Gradient methods by taking an initial search direction other than the direction $d_0 = -g_0$. In the present thesis we have taken the initial search direction slightly deflect from $d_0 = -g_0$ and with this deflected direction also we have succeeded to achieve a convergent solution.

In chapter 5 a new β has been proposed and the efficiency of this new proposed β is analysed in light of the experimental values.

From the investigation in Chapter 6, it is clear that if we take the value of the difference between the consecutive pair of step lengths from the sequence $\{\alpha_k\}$, we observe that the value of each pair is different from the value of the next consecutive pair for different values of $x_0, x_1, x_2, ...$ and $g_0, g_1, g_2, ...$ i.e. the sequence of step length tends to vary in a totally unpredictable manner.

From the above investigation and discussion in Chapter 4,Chapter 5 and Chapter 6 ,it is clear that the future prospect of the present work is open as the nature of γ needs more investigation.