TDC (CBCS) Even Semester Exam., 2022

PHYSICS

(Honours)

(4th Semester)

Course No. : PHSHCC–401T

(Mathematical Physics—III)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Define modulus and argument of a complex number.

2. Show that the sum and product of a complex number and its conjugate complex number are both real.

3. State and explain De Moivre's theorem.

4. What is singularity of an analytic function? Define pole.

5. State Taylor and Laurent expansions.

6. Expand $\cos z$ in a Taylor series about $z = \pi / 4$.

7. State Cauchy residue theorem.

8. How will you find the residue at a simple pole?

9. Find the residue at each pole of the function $f(z) = \cot z$.

10. Define Laplace transform of a function.

11. If $L\{f(t)\} = F(s)$, then show that

$$L\{f(at)\} = \frac{1}{a} F(s/a)$$

12. Show that Laplace transform of derivative of $f(t)$ corresponds to multiplication of the Laplace transform of $f(t)$ by s.

13. What is inverse Laplace transform?

14. Find the inverse Laplace transform of

$$\frac{1}{(s-2)^2 + 1}$$

15. State and explain convolution theorem.

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. *(a)* If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then prove that

$$x^p y^q + \frac{1}{x^p y^q} = 2\cos(p\theta + q\phi)$$ 3

(b) Find the square root of $-4-3i$. 3

17. *(a)* Show that the real and imaginary parts of an analytic function

$$f(z) = u(x, y) + iv(x, y)$$

satisfy the Cauchy-Riemann differential equations at each point where $f(z)$ is analytic. 4

(b) Show that $\sin z$ is analytic function of complex variable $z = x + iy$. 2

18. State and prove Cauchy's integral formula. 6

19. *(a)* Evaluate $\int_C \frac{dz}{z^2-1}$, where C is a circle $x^2+y^2=4$. 3

(b) Expand

$$f(z)=\frac{1}{(z+1)(z+3)}$$

as a Laurent series valid for *(i)* $|z|<1$ and *(ii)* $1<|z|<3$. 3

20. *(a)* Evaluate the residues of

$$\frac{z^2}{(z-1)(z-2)(z-3)}$$

at $z=1, 2, 3$ and infinity and show that their sum is zero. 3

(b) Find the residue of a function

$$f(z)=\frac{z^2}{(z+1)^2(z-2)}$$

at its double pole. 3

21. *(a)* Using Residue theorem, calculate

$$\frac{1}{2\pi i}\int_C \frac{e^{zt}dz}{z^2(z^2+2z+2)}$$

where C is the circle $|z|=3$. 3

(b) Using residue calculus, evaluate the following integral : 3

$$\int_0^{2\pi}\frac{1}{5-4\sin\theta}\,d\theta$$

22. *(a)* Find the Laplace transform of

$$F(t)=\begin{cases}1, & 0\le t<1\\ t, & 1\le t<2\\ t^2, & 2\le t<\infty\end{cases}$$ 3

(b) Show that Laplace transform of integral of $f(t)$, i.e.,

$$L\left[\int_0^t f(t)\,dt\right]=\frac{1}{s}F(s)$$

where $L[f(t)]=F(s)$. 3

23. *(a)* Find the Laplace transform of $t^2u(t-3)$. 3

(b) Find the Laplace transform of the function

$$f(t)=\begin{cases}\sin\omega t & \text{for } 0<t<\pi/\omega\\ 0 & \text{for } \pi/\omega<t<2\pi/\omega\end{cases}$$ 3

24. *(a)* Find the inverse Laplace transform of

$$\frac{(s+4)}{s(s-1)(s^2+4)}$$ 3

(b) Using the convolution theorem, calculate

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\};\ a \neq b$$ 3

25. (a) Solve the following differential equation using Laplace transform : 3

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

Given, $y(0)=2;\ y'(0)=0$.

(b) Using Laplace transforms, find the solution of the initial value problem

$$y''-4y'+4y=64\sin 2t;$$

$$y(0)=0,\ y'(0)=1$$ 3

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