2022/TDC (CBCS)/EVEN/SEM/
MTMDSE–602T (A/B)/267

TDC (CBCS) Even Semester Exam., 2022

MATHEMATICS

(6th Semester)

Course No. : MTMDSE–602T

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer from *either Option—A or Option—B*

OPTION—A

Course No. : MTMDSE–602T (A)

(Hydrodynamics)

SECTION—A

Answer any *twenty* of the following as directed : 1×20=20

1. Define real fluid and give two examples of real fluid.

2. Name the methods of describing fluid motion.

3. Under what condition are streamlines and pathlines coincide?

4. Velocity potential exists only when

(a) curl $\vec{q} = \vec{0}$

(b) div $\vec{q} = 0$

($\vec{q}$ is the fluid velocity)

(Choose the correct answer)

5. Is the motion given by $u = kx$, $v = 0$, $w = 0$ rotational?

6. What is the physical significance of the equation of continuity?

7. Write the equation of continuity for an incompressible fluid in vector form.

8. What is the equation of continuity in Lagrangian form?

9. Write the equation of continuity for any fluid in Cartesian coordinates.

10. Write the equation of continuity of any fluid in spherical polar coordinates.

11. Acceleration is the _____ derivative of fluid velocity.

(Fill in the blank)

12. Write the relation between material, local and convective derivative.

13. Write the components of acceleration of a fluid particle in Cartesian coordinates.

14. Define stream function.

15. Stream function exists in all types of _____ dimensional motion. Whether rotational or irrotational.

(Fill in the blank)

16. Write Euler's equation of motion in vector form.

17. What is Lamb's hydrodynamical equation?

18. Write Euler's equation of motion in Cartesian coordinates.

19. Euler's equation of motion expresses the principle of _____.

(Fill in the blank)

20. State the energy equation for a inviscid fluid.

21. Bernoulli's equation is obtained by _____ Euler's equation of motion.

(Fill in the blank)

22. If the motion is steady, velocity potential does not exist and V be the potential function from which the external forces are derivable, then Bernoulli's theorem is

(a) $-\frac{\partial \phi}{\partial t} + \frac{1}{2}q^2 + V + \int \frac{dp}{\rho} = C$

(b) $\int \frac{dp}{\rho} + \frac{1}{2}q^2 + V = C$

(c) $\frac{p}{\rho} + \frac{q^2}{2} + V = C$

(d) None of the above

(Choose the correct answer)

23. The Bernoulli's equation for unsteady and irrotational motion is given by

(a) $-\frac{\partial \phi}{\partial t} + \frac{q^2}{2} + V + \frac{p}{\rho} = F(t)$

(b) $-\frac{\partial \phi}{\partial t} + \frac{q^2}{2} + V = F(t)$

(c) $-\frac{\partial \phi}{\partial t} - \frac{q^2}{2} + V - \frac{p}{\rho} = F(t)$

(d) $\frac{q^2}{2} + V + \frac{p}{\rho} = F(t)$

(Choose the correct answer)

24. A stream in a horizontal pipe, after passing a contraction in the pipe at which the sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. If a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth h below the pipe, s being the delivery per second, where h is given by

(a) $\frac{s^2}{2g}\left(\frac{1}{A^2} + \frac{1}{B^2}\right)$

(b) $\frac{s^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$

(c) $\frac{2g}{s^2}\left(\frac{1}{A^2} + \frac{1}{B^2}\right)$

(d) $\frac{2g}{s^2}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$

(Choose the correct answer)

25. If the fluid be homogeneous and incompressible, Bernoulli's theorem becomes _____.

(Fill in the blank)

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Define streamlines and pathlines.

27. What do you mean by rotational and irrotational motion?

28. If the velocity components of an incompressible flow are given by

$$u = \frac{ax - by}{x^2 + y^2},\ v = \frac{ay + bx}{x^2 + y^2},\ w = 0,$$

then is the motion possible?

29. Given that

$$\frac{dJ}{dt} = J\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

where

$$J = \frac{\partial(x, y, z)}{\partial(a, b, c)}$$

Using this, derive Lagrangian form of equation of continuity from Eulerian form.

30. Determine the component of acceleration in x-direction at the point (2, 1, 3) at time $t = 0 \cdot 5$ sec, if $u = yz + t$, $v = xz - t$, $w = xy$.

31. If $u = 2Axy$, $v = A(a^2 + x^2 - y^2)$ are the velocity components of a fluid motion, determine the stream function.

32. Obtain Euler's equation of motion in Cartesian coordinates from its vector equivalent.

33. What do you mean by conservative force?

34. What is D'Alembert's paradox?

35. State Euler's momentum theorem.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. Describe the methods of describing fluid motion. 8

37. Determine the equations of streamlines for the flow specified by

$$\vec{q} = \frac{k^2(x\hat{j} - y\hat{i})}{x^2 + y^2} \quad (k = \text{constant})$$

Also test whether the motion is of potential kind and if so, determine the velocity potential. 3+5=8

38. Derive the equation of continuity in the form

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{q}) = 0$$ 8

39. Derive the equation of continuity in the form (in cylindrical polar coordinates)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho_r q_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho q_\theta) + \frac{\partial}{\partial z}(\rho q_z) = 0$$ 8

40. Define material, local and convective derivatives and establish the relation between them. 2+6=8

41. *(a)* Show that the stream function satisfies Laplace's equation in case of a two-dimensional irrotational motion. 4

(b) Determine the acceleration of a fluid particle for the velocity field given by

$$\vec{q} = (Axy^2t)\hat{i} + (Bx^2yt)\hat{j} + (Cxyz)\hat{k}$$ 4

42. Obtain Euler's equation of motion in the form

$$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p$$ 8

43. Obtain the equation of motion for an inviscid fluid in Lamb's hydrodynamical form. 8

44. State and prove Bernoulli's theorem. 8

45. A stream is rushing from a boiler through a conical pipe, the diameter of the ends of which are D and d, if V and v be the corresponding velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone, prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/2k}$$ 8

OPTION—B

Course No. : MTMDSE–602T (B)

(Theory of Equations)

SECTION—A

Answer any *twenty* of the following as directed : 1×20=20

1. What is the remainder when $3x^2 + 4x - 11$ is divided by $x - 1$?

2. State fundamental theorem of algebra.

3. What will be the nature of the roots if the signs of the terms of an equation be all positive?

4. State remainder theorem.

5. If $f(\alpha)$ and $f(\beta)$ be of opposite signs, then what can you say about the number of real roots between α and β of $f(x) = 0$?

6. Find the sum and product of the roots of the equation $4x^3 + 7x - 3 = 0$.

7. If α and β are the roots of $x^2 - 2x + 3 = 0$, then find the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.

8. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then find the value of k.

9. If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2$.

10. If the sum of the roots of the equation $\lambda x^2 + 2x + 3\lambda = 0$ be equal to their product, then find the value of λ.

11. Name any one method to solve a cubic equation.

12. Write down the standard form of a biquadratic equation.

13. Under what transformation the equation $ax^3 + 3bx^2 + 3cx + d = 0$ reduces to $Z^3 + 3HZ + G = 0$?

14. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\frac{1}{\alpha}$.

15. If α, β, γ and δ be the roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find the value of $\Sigma\alpha\beta$.

16. If all the roots of $f(x) = ax^3 + bx^2 + cx + d$ are real, then find the number of real roots of $f'(x)$.

17. Define superior limit of roots.

18. Find the number of imaginary roots of $x^5 + x^4 + x^2 - 25x - 36 = 0$.

19. Write the condition that the roots of the cubic equation $x^3 + 3Hx + G = 0$ should be real.

20. Let $f(x) = x^3 - 2x - 5$, find its first derived function $f_1(x)$.

21. Whether the equation $x^4 - 4x^3 + 8x + 4 = 0$ has commensurable roots?

22. Find the condition that the roots of the equation $ax^2 + 2bx + c = 0$ are real and unequal.

23. An equation in which the coefficient of the first term is unity, and the coefficients of the other terms are whole numbers, cannot have a commensurable root which is not a whole number.

(Write True or False)

24. Write the conditions that the roots of the cubic equation $Z^3 + 3HZ + G = 0$ are all real and unequal.

25. Horner's method is applied in solving any numerical equation to find both the commensurable and incommensurable roots.

(Write True or False)

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Find the quotient and remainder when $x^4+5x^3+4x^2+8x-20$ is divided by $x-1$.

27. Find the equation whose roots are $2, -3, 4, -1$.

28. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where p and q are real, then find (p, q).

29. If the difference of the roots of $x^2-px+8=0$ be 2, then find the value of p.

30. If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, then find the value of $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$.

31. State Newton's theorem on the sums of powers of roots.

32. Find all the roots of the equation $x^4-2x^3-19x^2+68x-60=0$ which lie between -6 and 6.

33. Find an approximate value of the positive root of the equation $x^3-2x-5=0$.

34. Find the integral roots of the equation $x^4-2x^3-13x^2+38x-24=0$.

35. Find all the commensurable roots of $2x^3-31x^2+112x+64=0$.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. (a) Express $3x^3-4x^2+5x+6$ as a polynomial in $x+1$. 4

(b) Prove that the equation $x^3+x^2-5x-1=0$ has one positive root lying in (1, 2) and two negative roots lying in $(-1, 0)$ and $(-3, -2)$. 4

37. (a) Apply Descarte's rule of signs to find the nature of the roots of the equation $x^4+qx^2+rx-s=0$ (q, r, s being positive). 4

(b) Solve the equation

$$x^4-3x^3-5x^2+9x-2=0, \ (2-\sqrt{3})$$

being one of its roots. 4

38. (a) If α, β, γ be the roots of the biquadratic equation $x^4+px^3+qx^2+rx+s=0$, then find the value of $\Sigma\alpha^4$. 4

(b) If α, β, γ be the roots of the equation $x^3 + 2x^2 + 1 = 0$, then find the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$. 4

39. (a) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ be the roots of the equation

$$x^n + \rho_1 x^{n-1} + \cdots + \rho_{n-1}x + \rho_n = 0, \rho_n \neq 0,$$

Find the value of $\Sigma \frac{\alpha_1^2 + \alpha_2^2}{\alpha_1 \alpha_2}$. 4

(b) Find the equation whose roots are the cubes of the roots of the equation $x^4 - 2x^3 + x^2 + 3x - 1 = 0$. 4

40. (a) Solve $x^3 - 18x - 35 = 0$ by Cardan's method. 4

(b) Solve the equation $x^4 - 2x^2 + 8x - 3 = 0$. 4

41. (a) Reduce the equation

$$x^3 + 6x^2 - 12x + 32 = 0$$

to its standard form and then solve the equation. 5

(b) If α, β, γ be the roots of the equation $x^3 + px + q = 0$, then find the value of $\Sigma \frac{1}{\alpha + \beta}$. 3

42. (a) Find the number and position of the real roots of the equation $x^4 - 2x^3 - 7x^2 + 10x + 10 = 0$. 4

(b) Apply Sturm's theorem to analyze the equation $x^4 - 4x^3 + 7x^2 - 6x - 4 = 0$. 4

43. (a) Calculate Sturm's functions for the following equation and show that four roots are imaginary : 4

$$3x^5 + 5x^3 + 2 = 0$$

(b) Prove that the roots of the equation $x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$ are all real and solve it when two of the quantities become equal. 4

44. (a) Find the positive root of the equation $x^3 + x^2 + x - 100 = 0$ correct to four decimal places. 4

(b) Find by Horner's method, the real positive root of the equation $8x^3 - 10x^2 - 3x - 7 = 0$ which lies between 1 and 2. 4

45. (a) Find a root of the equation $x^3 - 2x - 5 = 0$ correct to two places of decimal by Newton's method of approximation. 4

(b) Find in the form of a continued fraction the positive root of the equation $x^3 - 2x - 5 = 0$. 4

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