**2022/TDC (CBCS)/EVEN/SEM/**
**MTMDSE–601T (A/B/C)/266**

**TDC (CBCS) Even Semester Exam., 2022**

# MATHEMATICS

**( 6th Semester )**

Course No. : MTMDSE–601T

*The figures in the margin indicate full marks for the questions*

Candidates have to answer from *either* Option—A *or* Option—B *or* Option—C

## OPTION—A

Course No. : MTMDSE–601T (A)

**( Complex Analysis )**

*Full Marks* : 70
*Pass Marks* : 28

*Time* : 3 hours

## SECTION—A

Answer any *twenty* of the following questions : 1×20=20

**1.** Express $1+\sqrt{3}i$ in polar form.

**2.** What do you mean by Argand diagram?

3. Find the modulus and argument of $5+i4$.

4. Give a geometrical interpretation of

$$|z_1+z_2| \leq |z_1|+|z_2|$$

5. Show that amp $z = -\text{amp}\ \bar{z}$.

6. What do you mean by analytic function?

7. Define logarithmic function.

8. Give an example of a function which is continuous everywhere but nowhere differentiable.

9. Write down Cauchy-Riemann equation.

10. Define harmonic function.

11. State Cauchy-Goursat theorem.

12. When is a region said to be simply connected?

13. What do you mean by Jordan arc?

14. When is a curve said to be closed?

15. Define norm of a partition of $[a, b]$.

16. Define Taylor series.





17. What is the region of convergence in the Taylor series of the function $f(z) = \sin z$?

18. Find the zeroes of $f(z) = z^4 - z^2 - 3z + 3$.

19. State fundamental theorem of algebra.

20. What is the Taylor series expansion of $\log\left(\frac{1+z}{1-z}\right)$ at $z=0$?

21. State isolated singularities of a function.

22. Define simple pole.

23. What is mesomorphic function?

24. Give one example of an entire function.

25. Find the removable singularity of $\frac{\sin z}{z}$.

## SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Prove that $\arg z^n = n \arg z$.

27. Find the loci of the point $z$ satisfying

$$\arg \frac{z-1}{z+1} = \frac{\pi}{3}$$

**28.** Show that the function $e^x(\cos y + i\sin y)$ is harmonic.

**29.** Show that $u = x^3 - 3xy^2 - 3x^2 - 3y^2 + 1$ satisfies Laplace's equation.

**30.** Using the definition of an integral as the limit of a sum, calculate the integral

$$\int_L dz$$

**31.** Evaluate $\int_L \frac{dz}{z-a}$, where $L$ represents a circle $|z-a| = r$.

**32.** Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for the region $|z| < 1$.

**33.** State Maclaurin's series.

**34.** Define zeroes of an analytic function.

**35.** Find the residue of $\frac{1}{(z^2+1)^3}$ at $z = i$.





## SECTION—C

Answer any *five* of the following questions : 8×5=40

**36.** *(a)* Prove that the area of the triangle whose vertices are the points $z_1$, $z_2$, $z_3$ on the Argand diagram is

$$\sum \{(z_2 - z_3)|z_1|^2 / 4iz_1\}$$

Also show that the triangle is equilateral, if

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$ 5

*(b)* Determine the regions of Argand diagram defined by $|z^2 - 2| < 1$. 3

**37.** *(a)* Show that the triangles whose vertices are $z_1$, $z_2$, $z_3$ and $z_1'$, $z_2'$, $z_3'$ are equilateral, if

$$(z_1 - z_2)(z_1' - z_2') = (z_2 - z_3)(z_2' - z_3') = (z_3 - z_1)(z_3' - z_1')$$

4

*(b)* Show that if the points $z_1$, $z_2$, $z_3$, $z_4$ are concyclic, then the expression

$$\frac{(z_3 - z_1)(z_4 - z_2)}{(z_3 - z_2)(z_4 - z_1)}$$

is purely real. 4

**38.** *(a)* Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin although the Cauchy-Riemann equations are satisfied at that point. 4

*(b)* Show that an analytic function with constant modulus is constant. 4

**39.** *(a)* Find analytic function $w=u+iv$, where

$$u=e^{-x}(x^2-y^2)\cos y+2xy\sin y$$ 4

*(b)* Show that the function $f(z)=z^n$, where $n$ is a positive integer is an analytic function. 4

**40.** State and prove Cauchy integral formula. 2+6=8

**41.** Integrate $z^2$ along the straight line $OM$ and also along the path $OLM$ consisting of two straight line segments $OL$ and $OM$, where $O$ is the origin, $L$ is the point $z=3$ and $M$ is the point $z=3+i$. Hence show that the integral $z^2$ along the closed path $OLMO$ is zero. 8

**42.** *(a)* State and prove Liouville's theorem. 1+3=4

*(b)* Obtain the Taylor or Laurent series which represents the function

$$f(z)=\frac{1}{(1+z^2)(2+z)}$$

when $1<|z|<2$. 4





**43.** *(a)* Prove the fundamental theorem of algebra. 4

*(b)* Obtain the Taylor and Laurent series which represents the function

$$\frac{z^2-1}{(z+2)(z+3)}$$

in the region $|z|<2$. 4

**44.** *(a)* State and prove Cauchy's residue theorem. 5

*(b)* Evaluate the residues of

$$\frac{z^2}{(z-1)(z-2)(z-3)}$$

at $z=1, 2, 3$ and infinity and show that their sum is zero. 3

**45.** *(a)* Evaluate : 5

$$\int_0^{2\pi}\frac{d\theta}{(a+b\cos\theta)^2}, \quad (a>b>0)$$

*(b)* Find the the kind of the singularities of

$$\frac{\cot \pi z}{(z-a)^2}$$

at $z=0$ and $z=\infty$. 3

OPTION—B

Course No. : MTMDSE–601T (B)

**( Linear Programming )**

*Full Marks* : 70
*Pass Marks* : 28

*Time* : 3 hours

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. What type of LPP can be solved using graphical method?

2. What is a convex set in $\mathbb{E}^n$?

3. Introduce slack variable in the inequality

$$x_1 + 2x_2 + x_3 \le 7$$

and rewrite it accordingly.

4. What are surplus variables?

5. What is a convex polyhedron?





6. Find an initial basic feasible solution of the LPP :

$$\text{Max } Z = x + 2y + 3z$$
subject to
$$x - 3y + z \le 8$$
$$2x + y + 3z \le 7$$
$$x,\ y,\ z \ge 0$$

7. How should you modify the objective function of a minimization LPP in order to apply Simplex method?

8. Mention two methods to solve LPP using artificial variables.

9. What is the objective function in the first phase of the two-phase method?

10. In solving an LPP by Big-M method, state the condition under which it can be concluded that the LPP has no feasible solution.

11. Mention three types of primal-dual problem.

12. If the 4th variable in primal is unrestricted in sign, then what can you say about the 4th constraint in its dual?

13. If the dual has unbounded solution, then what can you conclude about the solution of the primal?

**14.** What is an unbalanced transportation problem?

**15.** Which cell gets the first allocation in North-West corner rule?

**16.** When is the solution of a transportation problem called degenerate?

**17.** What is the relation between $c_{ij}$, $u_i$ and $v_j$ for occupied cells?

**18.** Mention one basic assumption of an assignment problem.

**19.** Write True or False :

Assignment problem is a special type of transportation problem.

**20.** When is an assignment problem called unbalanced?

**21.** What is a payoff matrix?

**22.** What is a two-person zero-sum game?

**23.** State the minimax principle.

**24.** What is a symmetric game?

**25.** What is saddle point?





## SECTION—B

Answer any *five* of the following questions : 2×5=10

**26.** Justify with example that the union of two convex sets may not be convex.

**27.** Write the LPP in the standard form :

$$\text{Max } Z = 3x + 4y - z$$

subject to

$$x - 2y + 3z \le 4$$
$$2x + 3y + z \ge 5$$
$$x, y, z \ge 0$$

**28.** Write a short note on Big-M method.

**29.** Construct the auxiliary LPP of two-phase method for the LPP

$$\text{Max } Z = 5x_1 + 8x_2$$

subject to

$$3x_1 + 2x_2 \ge 3$$
$$x_1 + 4x_2 \ge 4$$
$$x_1, x_2 \ge 0$$

**30.** Write the dual of

$$\text{Max } Z = 3x + 4y + z$$

subject to

$$4x + 2y + z \le 3$$
$$2x + y + 3z \le 5$$
$$x, y, z \ge 0$$

**31.** Write a short note on North-West corner rule.

**32.** Explain loop in a transportation table.

**33.** State the assignment problem mathematically.

**34.** Write the analytical definition of saddle point.

**35.** Two boys *A* and *B* simultaneously draw either one or two ball(s) which they have in their bags. If the number of balls drawn by *B* be the same as the number of balls drawn by *A*, then *A* wins and gets one rupee from *B*. If the number of balls is not same, then *B* wins and gets one rupee from *A*. Write the payoff matrix of this game.

SECTION—C

Answer any *five* of the following questions : 8×5=40

**36.** *(a)* A manufacturing company produces two types of products *A* and *B*. The profit on each product is ₹ 5 and ₹ 7 respectively. The company must produce at least a total of 1000 products per month. However, the raw materials are sufficient for at most 400 products of type *A* per month. Each product of type *A* requires 2 hours and each product of type *B* requires 3 hours to





manufacture and the company has 25 working days each of 10 hours work time. Formulate the problem as an LPP so as to maximize profit. 3

*(b)* Solve graphically : 5

$$\text{Max } Z = 3x + 5y$$

subject to the constraints

$$3x + 2y \le 12$$
$$-x + y \le 3$$
$$y \le 4$$
$$x,\ y \ge 0$$

**37.** *(a)* Show that a convex polyhedron is a convex set. 5

*(b)* Explain the standard form of an LPP. Give an example to illustrate the same. 3

**38.** *(a)* Solve using Simplex method : 5

$$\text{Max } Z = 2x_1 + 5x_2 + 7x_3$$

subject to

$$3x_1 + 2x_2 + 4x_3 \le 100$$
$$x_1 + 4x_2 + 2x_3 \le 100$$
$$x_1 + x_2 + 3x_3 \le 100$$
$$x_1,\ x_2,\ x_3 \ge 0$$

*(b)* Write a note on two-phase method. 3

**39.** *(a)* Solve using Big-M method : 6

$$\text{Max } Z = -2x_1 - x_2$$

subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

*(b)* In the Big-M method, what conclusions can be drawn if some artificial variables are present in the basis but optimality conditions are satisfied? 2

**40.** *(a)* Write the dual of the following LPP : 4

$$\text{Max } Z = x + 2y + 3z$$

subject to

$$x + 3y - z \geq 8$$
$$2x + y \leq 5$$

where $x \geq 0$, $y$ is unrestricted in sign.

*(b)* Find an initial basic feasible solution of the following transportation problem by matrix minima method : 4

| Source | Destination $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $S_1$ | 2 | 3 | 1 | 10 |
| $S_2$ | 4 | 1 | 5 | 10 |
| $S_3$ | 6 | 2 | 7 | 15 |
| $S_4$ | 1 | 4 | 3 | 5 |
| Requirement → | 15 | 10 | 15 | |





**41.** *(a)* State and prove a necessary and sufficient condition for a transportation problem to have a feasible solution. 4

*(b)* Find an initial basic feasible solution of the following transportation problem using Vogel's approximation method : 4

| | $M_1$ | $M_2$ | $M_3$ | |
|---|---|---|---|---|
| $O_1$ | 2 | 7 | 4 | 5 |
| $O_2$ | 3 | 3 | 1 | 8 |
| $O_3$ | 5 | 4 | 7 | 7 |
| $O_4$ | 1 | 6 | 2 | 14 |
| | 7 | 9 | 18 | |

**42.** *(a)* Solve the following transportation problem so as to find its optimal solution : 6

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| $S_1$ | 2 | 3 | 11 | 7 | 6 |
| $S_2$ | 1 | 0 | 6 | 1 | 1 |
| $S_3$ | 5 | 8 | 15 | 9 | 10 |
| | 7 | 5 | 3 | 2 | |

*(b)* Write how you can resolve degeneracy in a transportation problem. 2

**43.** *(a)* Solve using Hungarian method : 6

| Man → | I | II | III | IV |
|---|---|---|---|---|
| 1 | 15 | 13 | 14 | 17 |
| 2 | 11 | 12 | 15 | 13 |
| 3 | 13 | 12 | 10 | 11 |
| 4 | 15 | 17 | 14 | 16 |

(b) How do you solve an unbalanced assignment problem? 2

**44.** (a) Solve the following game whose payoff matrix is given by

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | −5 | 2 | 1 | 6 |
| $A_2$ | 5 | 6 | 4 | 8 |
| $A_3$ | 4 | 0 | 1 | −3 |

5

(b) Show that the following payoff matrix has no saddle point : 3

| | | B | | |
|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ |
| | $A_1$ | 1 | 3 | 6 |
| A | $A_2$ | 2 | 1 | 3 |
| | $A_3$ | 6 | 2 | 1 |

**45.** (a) Solve graphically the game whose payoff matrix is

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | 2 | 2 | 3 | −1 |
| $A_2$ | 4 | 3 | 2 | 6 |

5

(b) Transform to LPP : 3

| | | B | |
|---|---|---|---|
| | 1 | −1 | −1 |
| A | −1 | −1 | 3 |
| | −1 | 2 | −1 |





OPTION—C

Course No. : MTMDSE–601T (C)

**( Object-oriented Programming in C++ )**

*Full Marks* : 50

*Pass Marks* : 20

*Time* : 3 hours

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

**1.** What is C++?

**2.** List the application of C++.

**3.** How is C++ different from C?

**4.** What is the structure of a C++ program?

**5.** What is a class?

**6.** Define the arrays.

**7.** Define a pointer.

**8.** What is dynamic binding?

**9.** Define an object.

**10.** What is class component?

**11.** What is assignment operator?

**12.** What do you mean by shallow coping?

**13.** What is class function?

**14.** What is class declaration?

**15.** What is friend function?

**16.** Write the purpose of resolution operator.

**17.** What are the arithmetic operators?

**18.** What is the use of friend function?

**19.** What is the syntax to overload an operator?

**20.** What is compile time in C++?

SECTION—B

Answer any *five* of the following questions : 2×5=10

**21.** Give the features of C that are not in C++.

**22.** Compare the OOP language and structured programming language.

**23.** How to declare a pointer to an object?





**24.** Differentiate between class and structure.

**25.** What are abstract classes and how are they different from a class?

**26.** What is the difference between a constructor and a destructor?

**27.** What is the default access mode for class members?

**28.** State the use of scope resolution operator in C++.

**29.** How is polymorphism achieved at run-time?

**30.** What is the difference between function overloading and function template?

SECTION—C

Answer any *five* of the following questions : 5×5=25

**31.** Explain the principles of object-oriented programming.

**32.** Briefly write about the evolution of C++.

**33.** Demonstrate encapsulation and polymorphism.

**34.** How is a pointer declared and initialized? Give an overview of pointer arithmetic.

**35.** What is a constructor? Write different rules associated with declaring constructors.

**36.** How to define a class in C++? How to declare objects for the class? Give an example.

**37.** Write a C++ program that declares and uses pointer to a class.

**38.** What is operator overloading? Illustrate with an example.

**39.** How does polymorphism promote extensibility? Illustrate.

**40.** Write C++ program to overload '+' operator to add two matrices.

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