TDC (CBCS) Even Semester Exam., 2022

MATHEMATICS

(Honours)

(4th Semester)

Course No. : MTMHCC-401T

(Numerical Methods)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Define absolute error and relative error.

2. Write an example of an ill-conditioned problem.

3. If $f(x) = x^2$, then find the value of $\Delta^3 f(x)$.

4. What is interpolation?

5. Why are polynomials used in numerical analysis?

6. Construct a forward difference table for $f(x) = x^3 + 2x + 1$ taking $x = 1, 2, 3, 4$.

7. Write the general quadrature formula for numerical integration.

8. What is the geometrical significance of Simpson's one-third rule?

9. What are the degrees of the approximating polynomials corresponding to trapezoidal rule and Simpson's three-eighth rule?

10. Mention the advantages of Newton-Raphson method over other methods.

11. When may the bisection method be used to find a root of the equation $f(x) = 0$?

12. Write the geometrical significance of regula-falsi method.

13. When is a matrix said to be diagonally dominant?

14. State the sufficient conditions for convergence of Gauss-Siedel method and Gauss-Jacobi method.

15. What is pivoting? Why is it important?

SECTION—B

Answer any *five* of the following questions : 6×5=30

16. *(a)* Define the operator E and show that $E^{-1} = 1 - \nabla$. 3

(b) Prove that $\Delta - \nabla = \Delta \cdot \nabla$. 3

17. *(a)* Show that ∇ is a linear operator. 3

(b) Prove that

$$\Delta \log f(x) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$$ 3

18. Establish Newton's forward-difference formula. When is the formula usually used? 5+1=6

19. Calculate the net premium at age 23 from the following table by using Lagrange's interpolation formula :

Age x	:	18	22	26	30
Premium f(x)	:	0·01527	0·01681	0·01872	0·02096

20. Deduce Simpson's $\frac{1}{3}$ rule for evaluating the integral

$$\int_a^b f(x)\,dx$$

21. Evaluate

$$\int_0^{\pi/2} \sqrt{\sin x}\,dx$$

taking six equal intervals, correct up to four significant figures by Simpson's $\frac{1}{3}$ rule and trapezoidal rule.

22. Find a root of the equation $x^x + 2x - 6 = 0$ by the bisection method correct to three decimals.

23. Describe regula-falsi method for finding the real root of an equation. Discuss the advantages of this method.

24. Solve the following by Gauss elimination method :

$$2x_1 + 3x_2 + x_3 = 9$$
$$x_1 + 2x_2 + 3x_3 = 6$$
$$3x_1 + x_2 + 2x_3 = 8$$

25. Solve the following by Gauss-Siedel iteration method :

$$x_1 + x_2 + 4x_3 = 9$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$

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