2022/TDC(CBCS)/EVEN/SEM/
MTMDSC/GEC–201T/258

TDC (CBCS) Even Semester Exam., 2022

MATHEMATICS

(2nd Semester)

Course No. : MTMDSC/GEC–201T

(Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *twenty* questions : 1×20=20

1. Define exact differential equation.

2. Define Clairaut's equation.

3. What is the integrating factor of the equation

$$2\frac{dy}{dx}+\frac{y}{x}=x^2?$$

4. Write the value of $d[\log_e(xy)]$.

5. If

$$\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(x)$$

what is the integrating factor of $Mdx + Ndy = 0$?

6. Write down the form of 2nd-order non-homogeneous linear differential equation.

7. Give an example of a 3rd-order homogeneous linear differential equation with constant coefficients.

8. Which is the non-homogeneous term of the following differential equation?

$$\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+x^3y=xe^{x+1}$$

9. State the condition under which the solutions of a linear differential equation are linearly dependent.

10. Write down the value of $W(x^2, 2x)$.

11. Solve :

$$\frac{d^2y}{dx^2}=\cos x$$

12. Solve :

$$(D^2+1)y=0$$

13. Find the CF of the differential equation

$$(D^2+2D+1)y=xe^x$$

14. Find the PI of the differential equation

$$(D+a)^2y=e^{ax}$$

15. Find $\frac{1}{D^2+4}\sin 2x$.

16. Solve :

$$\frac{dx}{x}=\frac{dy}{0}=\frac{dz}{-x}$$

17. Write down Cauchy-Euler equation.

18. State the condition for the integrability of the equation $Pdx + Qdy + Rdz = 0$.

19. Show that the equation

$$ydx + xdy + zdz = 0$$

is integrable.

20. Solve :

$$\frac{dx}{2x} = \frac{dy}{3y} = \frac{dz}{z}$$

21. What is the degree of the partial differential equation

$$2\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{1/2} ?$$

22. Write down the order of the partial differential equation

$$z\left(\frac{\partial z}{\partial x}\right) + \left(\frac{\partial z}{\partial y}\right) = x$$

23. State whether the following partial differential equation is linear or non-linear :

$$\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\frac{\partial z}{\partial x}$$

24. Fill in the blank :

Degree of the partial differential equation

$$y\left\{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right\} = z\left(\frac{\partial z}{\partial x}\right)$$

is ____.

25. Fill in the blank :

Order of the partial differential equation

$$\left\{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1\right\}(x - y) = 49\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

is ____.

SECTION—B

Answer any *five* questions : 2×5=10

26. Find the integrating factor of the differential equation

$$(1 - x^2)\frac{dy}{dx} + 2xy = x(1 - x^2)^{1/2}$$

27. Solve :

$$p^2 + p - 6 = 0,\ p = \frac{dy}{dx}$$

28. Verify that e^x, e^{-x} and e^{2x} are solutions of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

29. Define linear dependent and linear independent solutions.

30. Find the general solution of the differential equation

$$\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$$

31. Solve :

$$(D^2 - 2)^2 y = 0$$

32. Solve :

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

33. Solve :

$$(2x + y)dx + xdy + 2zdz = 0$$

(Given that the condition of integrability is satisfied.)

34. Form a partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$

35. Form a partial differential equation by eliminating arbitrary constants a and b from

$$z = (x + a)(y + b)$$

SECTION—C

Answer any *five* questions : 8×5=40

36. (a) Solve : 4

$$\left(\frac{x}{x^2 + y^2} + x^2 y\right)dy + \left(xy^2 - \frac{y}{x^2 + y^2}\right)dx = 0$$

(b) Solve : 4

$$y = (1 + p)x + ap^2 \text{ where } p = \frac{dy}{dx}$$

37. (a) Solve : 4

$$ydx - xdy + (1 + x^2)dx + x^2 \sin y dy = 0$$

(b) Solve : 4

$$y + px = p^2 x^4, \; p = \frac{dy}{dx}$$

38. (a) Prove that $\sin 2x$ and $\cos 2x$ are solutions of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 0$$

and that these solutions are linearly independent. 4

(b) Show that e^x and xe^x are linearly independent solutions of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

on the interval $-\infty < x < \infty$. Write the general solution of the given equation. 4

39. (a) Show that e^{2x} and e^{3x} are linearly independent solutions of

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

Find the solution $y(x)$ with the property $y(0)=0$ and $y'(0)=1$. 4

(b) Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine this differential equation. 4

40. (a) Solve : 3

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$$

(b) Given

$$\frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0$$

where a, b, g are constants and $x = a'$, $\frac{dx}{dt} = 0$ when $t = 0$.

Show that

$$x = a + (a'-a)\cos\left(\frac{\sqrt{g}}{b}\right)t$$ 5

41. (a) Solve : 3

$$(D^2 + 4D + 4)y = e^{2x} - e^{-2x}$$

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = \tan x$$

by the method of variation of parameters. 5

42. (a) Solve : 5

$$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\{\log(1+x)\}$$

(b) Solve : 3

$$\frac{dx}{dt} + ky = 0$$
$$\frac{dy}{dt} - kx = 0$$

43. (a) Solve : 3

$$\frac{dx}{xz(z^2 + xy)} = \frac{dy}{-yz(z^2 + xy)} = \frac{dz}{x^4}$$

(b) Show that the differential equation

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

is integrable and hence solve it. 5

44. (a) Eliminate the arbitrary constants a and b from

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

to form a partial differential equation. 4

(b) Form a partial differential equation by eliminating the function f from

$$z = x^n f\left(\frac{y}{x}\right)$$ 4

45. (a) Form the partial differential equation by eliminating h and k from

$$(x-h)^2 + (y-k)^2 = \lambda^2$$ 4

(b) Eliminate the arbitrary function f from $z = f(x^2 - y^2)$ to form a partial differential equation. 4

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