

**2022/TDC(CBCS)/EVEN/SEM/  
MTMHCC-202T/257**

**TDC (CBCS) Even Semester Exam., 2022**

**MATHEMATICS**

**( Honours )**

**( 2nd Semester )**

**Course No. : MTMHCC-202T**

**( Differential Equations )**

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

**SECTION—A**

Answer any ten questions :

2×10=20

1. Write the degree and order of the following differential equations :

(i)  $y = \sqrt{x} \frac{dy}{dx} + \frac{K}{\frac{dy}{dx}}$

(ii)  $\left( \frac{d^2y}{dx^2} \right)^{1/3} = \left( y + \frac{dy}{dx} \right)^{1/2}$

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2. Find the differential equation of all circles touching the X-axis at the origin.
3. Define Wronskian of  $n$  functions.
4. Find  $\alpha$  and  $\beta$  so that the equation  
 $(\alpha xy^3 + y \cos x)dx + (x^2 y^2 + \beta \sin x)dy$   
 is exact.
5. Solve :  
 $(1 + y^2)dx + (1 + x^2)dy = 0$
6. Find the integrating factor of  
 $\cos x \frac{dy}{dx} + y \sin x = 1$
7. Write the differential equation for drug assimilation pill in case of—  
 (a) single pill;  
 (b) course of pill.
8. Write two assumptions for limited growth with Harvesting Model.
9. What do you mean by Simple Compartmental Model?

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10. Solve :

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2 y^2 z^2}$$

11. What is the necessary and sufficient condition for integrability of the total differential equation  $Pdx + Qdy + Rdz = 0$ ?

Hence show that

$$zdx + zdy + 2(x + y + \sin z)dz = 0$$

is integrable.

12. Solve :

$$yz \log z dx - zx \log z dy + xy dz = 0$$

(Given that the condition of integrability is satisfied)

13. Solve :  $(D^2 + D)y = 0$

14. Find the CF of

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2e^{3x}$$

15. Find the PI of  $(D^2 - a^2)y = e^{ax}$ .

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## SECTION—B

Answer any five questions :

6×5=30

16. (a) Find the differential equation whose solution is the family of curves defined by  $x^2 + y^2 + 2ax + 2by + c = 0$  where  $a, b, c$  are parameters. 3

- (b) Prove that  $e^x$ ,  $e^{-x}$  and  $e^{2x}$  are the linearly independent solutions of

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0 \quad 3$$

17. (a) Form the differential equation for the curve  $y = k \sin^{-1} x$ ,  $k$  being a constant. 2

- (b) (i) Show that  $y = 2 \sin x + 3 \cos x$  is an explicit solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0$$

for all real  $x$ . 2

- (ii) Show that the solutions  $\sin x$  and  $\cos x$  of

$$\frac{d^2y}{dx^2} + y = 0$$

are linearly independent. 2

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18. (a) If the equation  $Mdx + Ndy = 0$  is homogeneous and  $Mx + Ny \neq 0$ , then show that  $\frac{1}{Mx + Ny}$  is an integrating factor. 3

- (b) Solve :  $\frac{dy}{dx} + \frac{y}{x} = x^2$   
given  $y = 1$  when  $x = 1$ . 3

19. (a) Solve :  $(6x - 5y + 4)dy + (y - 2x - 1)dx = 0$  3

- (b) Solve :  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$  3

20. Discuss the population growth model. 6

21. At time  $t = 0$ , a tank contains 4 lb of salt dissolved in 100 gal of water. Suppose that brine containing 2 lb of salt per gallon of brine is allowed to enter the tank at the rate of 5 gal/min and that the mixed solution is drained from the tank at the same rate. Find the amount of salt in the tank after 10 minutes. 6

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22. (a) Solve :

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} \quad 3$$

(b) Solve :

$$(2xy + z^2)dx + (x^2 + 2yz)dy + (y^2 + 2xz)dz = 0 \quad 3$$

23. (a) Solve :

$$\frac{dx}{dt} = x - 2y, \quad \frac{dy}{dt} = 5x + 3y \quad 4$$

(b) Test the integrability of the total differential equation

$$(2x^3y + 1)dx + x^4dy + x^2 \tan z dz = 0 \quad 2$$

24. (a) Solve :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4 \quad 3$$

(b) Solve :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left( x + \frac{1}{x} \right) \quad 3$$

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25. (a) Solve :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

Given that when  $x = 0$ ,  $y = 3$  and  $\frac{dy}{dx} = 0$ . 3

(b) Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = (x+1)e^{2x}$$

by the method of variation of parameters. 3

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