2020/TDC(CBCS)/ODD/SEM/
PHSHCC–301T/150

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

PHYSICS

(3rd Semester)

Course No. : PHSHCC–301T

(Mathematical Physics—II)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) State orthogonality conditions of sine and cosine functions.

(b) State the Fourier series theorem of a function $f(x)$ and write the Fourier coefficients.

(c) Find the Fourier coefficients when the function $f(x)$ is even.

(d) Write the complex form of the Fourier series.

(e) What do you mean by power series? State its conditions of convergence.

(f) Check if $x = 0$ is an ordinary point or singular point for the following differential equations :

(i) $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2 + 2)y = 0$

(ii) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (1 - x)y = 0$

(g) State the conditions for which $x = x_0$ be regular singular and irregular singular points for the differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

(h) State Bessel's differential equation of second order and write the expression for Bessel's function of first kind of order two.

(i) Use the generating function of $J_n(x)$ to find the values of $J_0(x)$ and $J_1(x)$.

(j) Show that $P_n(1) = 1$.

(k) Express $5x^3 - x + 2$ in terms of Legendre's polynomials.

(l) If (r_1, θ_1) and (r_2, θ_2) be the polar co-ordinates of any two points and $\theta = \theta_1 \sim \theta_2$, then show that the reciprocal of the distance between the two points is given by

$$\sum_{n=0}^{\infty} \frac{r_2^n}{r_1^{n+1}} P_n(\cos\theta)$$

(m) Show that

$$\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n+1)}{n} = \frac{\beta(m, n)}{m+n}$$

(n) Show that

$$\beta(m, n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1} d\theta$$

(o) Give the mathematical definition of Dirac-delta function.

(p) Evaluate :

(i) $\int_{-\infty}^{+\infty} x\delta(x - a)dx$

(ii) $\int_{-1}^{+1} 2\delta(x - 2)dx$

(q) Write the order and degree of the following differential equations :

(i) $\left(\frac{\partial^2 y}{\partial x^2}\right)^3 + \frac{\partial y}{\partial t} = 0$

(ii) $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y^2} = 0$

(r) Solve the differential equation

$$\frac{\partial^2 z}{\partial x \partial y} = x^2 y$$

(s) Write the Laplace equation in 2D cylindrical co-ordinate system.

(t) Write the Laplace equation in 2D spherical co-ordinate system.

SECTION—B

Answer *any* **five** questions

2. A periodic function of period 2π is defined as

$$f(x) = x^2, \ -\pi \le x \le \pi$$

Expand $f(x)$ in Fourier series and hence show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

3+3=6

3. A square wave of period T is defined by the function $f(t)$ as

$$f(t) = a \text{ for } t = 0 \text{ to } \frac{T}{2}$$
$$= 0 \text{ for } t = \frac{T}{2} \text{ to } T$$

Find the Fourier series of the function $f(t)$. 6

4. Solve by power series method, Legendre's differential equation

$$(1 - x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

in descending powers of x. 6

5. Use Frobenius method to solve Hermite differential equation

$$\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 2xy = 0$$

6

6. If a and b are different roots of $J_n(x) = 0$, then show that

$$\int_0^1 x J_n(ax) J_n(bx) = 0 \text{ for } a \ne b$$
$$= \frac{1}{2}[J_n'(a)]^2 \text{ for } a = b$$

6

7. Show that

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$$ 6

8. Show that

$$\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{\frac{1}{n}}} dy$$

Hence show that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$ 2+4=6

9. *(a)* Show that

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$ 2

(b) If $G_a(x)$ is the Gaussian function given by $G_a(-x) = \frac{a}{\sqrt{\pi}} e^{-a^2x^2}$, then show that

$$\delta(x) = \underset{a\to\infty}{\text{Lt}} \frac{a}{\sqrt{\pi}} e^{-a^2x^2} = \underset{a\to\infty}{\text{Lt}}\ G_a(x)$$ 4

10. *(a)* Solve the differential equation

$$\frac{\partial^2 z}{\partial x \partial y} = \cos(2x+3y)$$ 2

(b) Solve the following differential equation by the method of separation of variables :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$$ 4

11. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

under the following conditions :

$u(0, t) = 0$ and $u(l, t) = 0$ 6

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