**2019/TDC/ODD/SEM/
PHSHCC–101T/069**

TDC (CBCS) Odd Semester Exam., 2019

PHYSICS

(1st Semester)

Course No. : PHSHCC–101T

(Mathematical Physics—I)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Explain transpose of a matrix with an example.

(b) Show that any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

(c) If A and B are non-singular matrices of same order, then show that

$$(AB)^{-1} = B^{-1}A^{-1}$$

2. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$ 3

(b) Solve the following equations by matrix method : 3

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

(c) Show that

$$(3x + 4y + 5)\,dx + (4x - 3y + 3)\,dy = 0$$

is an exact equation and hence solve it. 3

(d) Solve the differential equation : 3

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$$

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) If $\vec{A} + \vec{B} + \vec{C} = 0$, show that

$$(\vec{A} \times \vec{B}) = (\vec{B} \times \vec{C}) = (\vec{C} \times \vec{A})$$

(b) Find the value of a such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar.

(c) Show that $\vec{\nabla}\phi$ is a vector normal to the surface $\phi(x, y, z) = C$.

4. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) For any three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, show that

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A})$$ 3

(b) Show that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$ 3

(c) A vector field is defined as

$$\vec{A} = \frac{\vec{r}}{r^2}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Evaluate $\vec{\nabla} \cdot \vec{A}$ and $\vec{\nabla} \times \vec{A}$ and hence state whether the field is solenoidal or irrotational.

(1½×2)+1=4

(d) Show that

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{C} \times (\vec{A} \times \vec{B}) + \vec{B} \times (\vec{C} \times \vec{A}) = 0$$ 2

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) Calculate the volume integral of $(\vec{\nabla} \cdot \vec{r})$ over the volume enclosed by a sphere of radius a, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(b) If $\vec{F} = \vec{\nabla}\phi$ everywhere in a region R and ϕ is single-valued and has continuous derivatives in R, then show that

$$\int_A^B \vec{F} \cdot d\vec{r}$$

is independent of the path joining the points A and B.

(c) Calculate the work done when a force $\vec{F} = 3xy\hat{i} - y^2\hat{j}$ moves a particle in xy-plane from (0, 0) to (1, 2) along the parabola $y = 2x^2$.

6. Answer *(a)* or *(b)* : 6

(a) Verify Gauss divergence theorem for the vector $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ over the surface of a cube bounded by $0 \le x, y, z \le 1$.

(b) State and prove Stokes' theorem.

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Write the expressions for line element and volume element in cylindrical coordinates.

(b) Write the expression for divergence of a vector in orthogonal curvilinear coordinates.

(c) Write the expression for Laplacian of a scalar in orthogonal curvilinear coordinates.

8. Answer *(a)* or *(b)* : 6

(a) Derive the expression for curl of a vector in orthogonal curvilinear coordinates.

(b) Describe the spherical coordinate system with necessary diagram to derive the expression for divergence of a vector in spherical coordinates.

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) What is meant by probability? Write the expression for probability function for binomial distribution. 1+1=2

(b) Explain the term 'variance' with an example.

(c) The following set of data gives the diameter of a wire measured by a screw gauge for five different observations :

2·12 cm, 2·11 cm, 2·14 cm, 2·13 cm, 2·15 cm

Find the standard deviation.

10. Answer *(a)* or *(b)* : 6

(a) Derive the expressions for mean and variance for Poisson distribution.

(b) What is conditional probability? State and prove Bayes' theorem.

★★★

2019/TDC/ODD/SEM/PHSHCC-102T/070

TDC (CBCS) Odd Semester Exam., 2019

PHYSICS

(1st Semester)

Course No. : PHSHCC-102T

(Mechanics)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Differentiate between inertial and non-inertial frames.

(b) Show that Newton's second law of motion is invariant under Galilean transformation.

(c) What are meant by laboratory and centre of mass frames of references?

2. Answer either [*(a), (b)*] or [*(c), (d)*] :

(a) Establish the following relation for rocket motion

$$v = v_0 + u \log_e M_0 / M$$

where v_0 is the initial velocity, u is the exhaust velocity of gases relative to rocket and M_0 is the initial mass. 3

(b) Prove that

$$\vec{J} = \vec{J}_{CM} + \vec{R} \times \vec{P}$$

where $\vec{J}$ = angular momentum in lab frame

$\vec{J}_{CM}$ = angular momentum in centre of mass frame

$\vec{P}$ = linear momentum of the system

$\vec{R}$ = position vector of centre of mass 3

(c) Neglecting the attraction of the earth, show that

$$v = v_0 - u \log_e \left(1 - \frac{mt}{M_0}\right)$$

where v_0 is the initial velocity, v is the velocity acquired in t sec, u is the exhaust velocity and m is the constant rate of burning of the fuel of a rocket. 3

(d) Show that in absence of any external force, the velocity of centre of mass of a system remains constant. 3

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) State the theorems of parallel and perpendicular axes with reference to moment of inertia.

(b) Write the expressions for torsional rigidities of a solid cylinder and a hollow cylinder.

(c) Show that the profile of a liquid flowing through a capillary tube is parabolic.

4. Answer either [*(a), (b)*] or [*(c), (d)*] :

(a) What are Newton's laws of viscous flow? Derive an expression for rate of flow of a liquid through capillaries in series. 1+2=3

(b) Prove that for an elastic medium

$$Y = \frac{9\eta K}{3K + \eta}$$

where the symbols have their usual meanings. 3

(c) What must be the relation between length l and radius R of a cylinder if the moment of inertia about the axis is to be same as its moment of inertia about the equatorial axis? 3

(d) Calculate the velocity of water along the axis of a capillary tube through which it is flowing under a pressure difference of 10 cm of water. The length of the capillary tube is 1 m and its diameter is 1 mm. The coefficient of viscosity of water is 0·001 kg/m s. 3

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Show that the principle of conservation of angular momentum applied to planetary motion leads to the law of constant areal velocity.

(b) What are the values of potential energy of a body taking reference level at *(i)* the earth's surface and *(ii)* infinity?

(c) Define escape velocity and geosynchronous orbit.

6. Answer either [*(a)*, *(b)*] or [*(c)*, *(d)*] :

(a) Show that for an external point a spherical shell behaves as if whole mass is concentrated at the centre deriving the expression for gravitational potential due to a spherical shell outside the shell. 3

(b) The mass of the moon is about 0·013 times the mass of the earth and the distance of centre of the moon from centre of the earth is about 60 times of radius of the earth. Find the distance of centre of mass of earth-moon system from the centre of the earth. Take radius of the earth is 6400 km. 3

(c) Show that the gravitational potential due to a solid sphere, inside the sphere is 3/2 times that on the surface of the sphere. 3

(d) Two bodies of masses m_1 and m_2 are interacting through a central force. Show that their equation of motion can always be reduced to single body equation. 3

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) What is the affect of Coriolis force on a freely falling body at the earth?

(b) A particle is executing SHM in air. Name the type of oscillation and write the differential equation of motion.

(c) Define bandwidth of resonance. What is the relation between bandwidth and relaxation time?

8. Answer either [*(a)*, *(b)*] or [*(c)*, *(d)*] :

(a) Amplitude of forced vibration is given by

$$A = f_0[(\omega_0^2 - p^2)^2 + p^2/\tau^2]^{-1/2}$$

where the symbols have their usual meanings. Discuss with the help of suitable graph the variations of A with ω for *(i)* low τ and *(ii)* high τ. 3

(b) Write a note on 'fictitious force'. 3

(c) Calculate the deviation of a freely falling body from a height of 100 m at latitude 45 °N due to Coriolis acceleration. 3

(d) What is resonance? Define sharpness of resonance and quality factor. 3

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Define proper time interval. Explain why a moving clock appears to go slow to a stationary observer.

(b) Discuss the motivation behind Michelson-Morley experiment.

(c) Show that whatever velocity be combined with velocity of light, the resultant is always the velocity of light.

10. Answer either [*(a)*, *(b)*] or [*(c)*, *(d)*] :

(a) A particle has a lifetime of 10^{-7} s when measured at rest. How far does it go before decaying if its speed is $0{\cdot}99c$ when created? 3

(b) Show that the rest mass of a particle of momentum p and kinetic energy T is

$$m = p^2c^2 - E^2/2Tc^2$$ 3

(c) Discuss briefly one experiment in support of time dilation in STR. 3

(d) State the fundamental postulates of STR. Explain the variation of mass with velocity with the help of a graph. 3

★★★