2020/TDC(CBCS)/ODD/SEM/
MTMSEC–501T/333D

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

MATHEMATICS

(5th Semester)

Course No. : MTMSEC–501T

(Integral Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *fifteen* questions : 1×15=15

1. Write down the value of

$$\int \frac{f'(x)}{f(x)}\,dx$$

2. Write down the value of

$$\int \frac{dx}{x^2+a^2}$$

3. If

$$\frac{d}{dx}[f(x)] = F(x)$$

then find the value of $\int F(x)\,dx$.

4. Find the value of

$$\frac{d}{dx}[\int f(x)\,dx]$$

5. Write down the value of $\int a^x dx$.

6. Write down the value of

$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$

7. If $f(x)$ is even, then what is the value of $\int_{-a}^{a} f(x)\,dx$?

8. Write down the value of

$$\int_{-\pi/2}^{\pi/2} \sin^3 x\,dx$$

9. Write True or False :

$$\int_0^{\pi/2} \log(\sin x)\,dx = \int_0^{\pi/2} \log(\cos x)\,dx$$

10. Find

$$\int_{-1}^{0} |x|\,dx$$

11. Find the value of

$$\int_{-a}^{a} x\phi(x^2)\,dx$$

12. Express $\int_a^b f(x)\,dx$ as limit sum.

13. Write down the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, when n is odd.

14. Write down the reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, when n is even.

15. Write True or False :

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx$$

16. Find the value of $\int_0^{\pi/2} \sin^4 x\,dx$ by using reduction formula.

17. If $\phi(n) = \int_0^{\pi/4} \tan^n x\,dx$, what is the value of $\phi(n) + \phi(n-2)$?

18. Write down the reduction formula for $\int \sec^n x\,dx$.

19. Write down the geometrical interpretation of $\int_a^b f(x)\,dx$.

20. Write down the parametric equation of circle $x^2 + y^2 = r^2$.

21. Write down the parametric equation of astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

22. Write down the formula of length in Cartesian form.

23. Write down the formula of length in parametric form.

24. What is the length of circumference of a circle of radius a?

25. What is the surface area of the sphere of radius a?

26. What is the volume of the solid generated by the curve $y = f(x)$, intercepted between $x = a$ and $x = b$ and the axis of revolution about x-axis?

(Continued)

27. What is the surface area of the solid generated by the curve $x = f(y)$, intercepted between $y = a$ and $y = b$ and the axis of revolution about y-axis?

28. What is the volume of the sphere generated by the rotation of the circle $x^2 + y^2 = 4a^2$?

29. What is the volume of the solid generated by the revolution of the area bounded by the curve $r = f(\theta)$ and radii vectors $\theta = \theta_1$, $\theta = \theta_2$ and revolution about initial line $\theta = 0$?

30. What is the volume of a paraboloid of revolution formed by revolving the parabola $y^2 = 4ax$ about the x-axis and bounded by the section $x = h$?

SECTION—B

Answer any *five* questions : 2×5=10

31. Evaluate :

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

32. Evaluate :

$$\int \frac{e^{\sqrt{x}}\cos(e^{\sqrt{x}})}{\sqrt{x}}\,dx$$

(Turn Over)

33. Prove that

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

34. Prove that

$$\int_0^{\pi/2} \log(\tan\theta)\,d\theta = 0$$

35. If $I_n = \int_0^{\pi/2} \sin^n x\,dx$, $n > 1$, then prove that

$$I_n = \frac{n-1}{n} I_{n-2}$$

36. If $I_n = \int \tan^n x\,dx$, $n > 1$, then prove that

$$I_n = \frac{\tan^n x}{n-1} - I_{n-2}$$

37. Find the length of the curve $x = e^\theta \sin\theta$, $y = e^\theta \cos\theta$; $\theta = 0$ and $\theta = \pi/2$.

38. Find the length of the curve $r = \theta^2$; $\theta = 0$ and $\theta = \sqrt{5}$.

39. Find the volume generated by revolving about x-axis, the area bounded by $y = \cos x$ between $x = 0$ and $x = \pi/2$.

40. Find the surface area generated by revolving about y-axis, the area bounded by $y = x^2$ between $y = 0$ and $y = \sqrt{2}$.

SECTION—C

Answer any *five* questions : 5×5=25

41. Evaluate :

$$\int (\sqrt{\tan x} + \sqrt{\cot x})\,dx$$

42. Evaluate :

$$\int \frac{dx}{x^4 + a^2}$$

43. Prove that

$$\int_0^{\pi/2} \log(\sin x)\,dx = \frac{\pi}{2}\log\frac{1}{2}$$

44. Evaluate :

$$\lim_{n\to\infty}\left[\left(1+\frac{1^2}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\cdots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$$

45. If $u_n = \int_0^{\pi/2} x^n \sin x\,dx$, $n > 1$, then prove that $u_n + n(n-1)u_{n-2} = n(\frac{\pi}{2})^{n-1}$.

46. If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\,dx$; m, n being positive integers greater than 1, then prove that

$$I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$$

47. Find the total length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

48. Find the length of an arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$.

49. Find the area of the surface generated by the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x-axis.

50. Find the volume of the ellipsoid by the revolution of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about the major axis.

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