2020/TDC (CBCS)/ODD/SEM/ MTMDSE–502T (A/B)/333 C

TDC (CBCS) Odd Semester Exam., 2020 held in March, 2021

MATHEMATICS

(5th Semester)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* from Option—A *or* from Option—B

OPTION—A

Course No. : MTMDSE–502T (A)

(ANALYTICAL GEOMETRY)

SECTION—A

Answer any *twenty* of the following as directed : 1×20=20

1. When the equation $ax^2+2hxy+by^2=0$ represents a pair of imaginary straight lines?

2. Write down the equation of bisectors of the angle between the pair of lines of $ax^2+by^2+2hxy=0$.

3. Write the formula for transformation of coordinates in case of rotation of axes.

4. What do you mean by a homogeneous equation of second degree in x and y?

5. Write down the transformed equation of the curve $x^2 - y^2 = 4$ when the axes are rotated through an angle 30°.

6. What is the angle between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$?

7. What is the condition that the lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are parallel to each other?

8. Write the condition that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

may represent a pair of parallel straight lines.

9. Write down the parametric equation of a parabola.

10. What do you mean by locus of a point?

11. How many tangents can be drawn from a given point to a parabola?

12. Define auxiliary circle.

13. What is the standard equation of a hyperbola?

14. Define conjugate hyperbola.

15. What is the condition of tangency of a line $y = mn + c$ to a circle $x^2 + y^2 = a^2$?

16. Define orthogonal circles.

17. Define pole of a polar.

18. Write down the equation of polar of the point (x_1, y_1) with respect to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

19. Is the statement, "If the polar of P with respect to conic passes through Q, then the polar of Q also passes through P" true?

20. Discuss the nature of the conic

$$\frac{15}{r} = 3 - 4\cos\theta$$

21. Define conic section.

22. What do you mean by eccentricity of a conic?

23. Write down the equation of directrix of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

24. What is the eccentricity of the conic

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \text{ ?}$$

25. Define shortest distance between two lines.

26. Define great circle.

27. What do you mean by sphere?

28. What is the shortest distance between two parallel lines?

29. Write down the centre and radius of the sphere $x^2 + y^2 + z^2 + 2x - 4y + 2z - 3 = 0$.

30. When a plane cuts a sphere the section is ______.

(Fill in the blank)

31. Write down the equation of the plane in intercept form.

32. What are the direction ratios of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) ?

33. Define cone.

34. What do you mean by a right circular cylinder?

35. What should be the value of l, m, n if the generators are parallel to z-axis?

36. Define right circular cone.

37. What type of equation represents a cone with vertex at origin?

38. What is cylinder?

39. Write down the equation of a cylinder.

40. What do you mean by slant height of a cone?

SECTION—B

Answer any *five* of the following questions : 2×5=10

41. Transform the equation $5x+6y=1$ to parallel axes through the new origin (1, 1).

42. Find the value of λ for which the equation $2x^2+3xy-2y^2+7x+y+\lambda=0$ may represent a pair of straight lines.

43. If l and l' are the lengths of two segments of focal chord, then prove that

$$\frac{1}{l}+\frac{1}{l'}=\frac{1}{a}$$

44. The normal at the point $(at_1^2, 2at_1)$ meets the parabola $y^2=4ax$ again at the point $(at_2^2, 2at_2)$. Prove that

$$t_2=-t_1-\frac{2}{t_1}$$

45. Find the point on the curve $\frac{14}{r}=3-8\cos\theta$ whose radius vector is 2.

46. Find the pole of the line $lx+my+n=0$ with respect to the parabola $y^2=4ax$.

47. Find the equation of the sphere through the circle $x^2+y^2+z^2=5$, $x+2y-z+2=0$ and the point (1, 1, 1).

48. Find the shortest distance between x-axis and the line

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

49. Draw a cone and mark its vertex, height and slant height.

50. Prove that the direction cosines of a generator of a cone whose vertex is origin satisfy the equation of the cone.

SECTION—C

Answer *any* **five** of the following questions : 8×5=40

51. *(a)* Prove that the product of perpendicular from the point (x', y') on the line $ax^2+2hxy+by^2=0$ is

$$\frac{ax'^2+2hx'y'+by'^2}{\sqrt{(a-b)^2+4h^2}}$$ 4

(b) Prove that the pair of straight lines joining the origin to the other two points of intersection of the

curve $ax^2+2hxy+by^2+2gx=0$ and $a'x^2+2h'xy+b'y^2+2g'x=0$ will be at right angles if $g(a'+b')=g'(a+b)$. 4

52. (a) If by changing the axes (the sets of axes being rectangular) but without change of origin the expression $ax^2+2hxy+by^2$ becomes $a'x^2+2h'xy+b'y^2$, then show that $a+b=a'+b'$ and

$$ab-h^2=a'b'-h'^2$$ 4

(b) Find the angle through which the axes to be rotated in order to remove the term containing xy from the expression $ax^2+2hxy+by^2$. 4

53. (a) Show that any three normals can be drawn from a given point to a parabola. Hence show that the sum of the ordinates of three conormals is zero. 4

(b) Show that the product of the length of the perpendicular drawn from the foci on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is b^2. 4

54. (a) Find the equation of the circle orthogonal to both the circles $x^2+y^2+3x-5y+6=0$ and $4x^2+4y^2-28x+29=0$ and whose centre lies on the line $3x+4y+1=0$. 4

(b) Show that two tangents can be drawn from a given point in a circle. 4

55. (a) Find the polar equation of a conic of the form $\frac{l}{r}=1+e\cos\theta$. Write down the equation of directrix. 4

(b) The pole of the straight line with respect to the circle $x^2+y^2=a^2$ lies on $x^2+y^2=k^2a^2$. Prove that the straight line will touch the circle

$$x^2+y^2=\frac{a^2}{k^2}$$ 4

56. (a) A conic is described having the same focus and eccentricity as the conic $\frac{l}{r}=1+e\cos\theta$ and the two conics touch at $\theta=\alpha$. Prove that the length of its latus rectum is

$$\frac{2l(1-e^2)}{e^2+2e\cos\alpha+1}$$ 4

(b) The polar of the point P with respect to the circle $x^2 + y^2 = a^2$ touches the circle $4x^2 + 4y^2 = a^2$. Show that the locus of P is the circle $x^2 + y^2 = 4a^2$. 4

57. *(a)* Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}; \quad \frac{x-3}{4} = \frac{y-5}{5} = \frac{z-7}{1}$$

Also find the equation of the line of shortest distance. 4

(b) A plane passes through a fixed point (a, b, c) and cuts the sphere $OABC$ is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

4

58. *(a)* Find the equation of sphere touching the three coordinate planes. How many such spheres can we have? 4

(b) Find the length and equation of shortest distance between the line

$$x + y + 2z - 3 = 0 = 2x + 3y + 4y - 4$$

and z-axis. 4

59. *(a)* Find the equation of the cone whose vertex is at $(1, 2, 3)$ and guiding curve is the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$. 4

(b) Find the equation of the cylinder generated by the lines parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{5}$, the guiding curve being the conic $x = 0$, $y^2 = 8z$. 4

60. *(a)* Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular generators if

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

4

(b) Find the equation of a right circular cone generated when the straight line $2y + 3z = 6$, $x = 0$ revolves about z-axis. 4

OPTION—B

Course No. : MTMDSE–502T (B)

(Probability and Statistics)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. Given $E(X) = 5$, $E(Y) = -2$. Find $E(X - Y)$.

2. If X is a random variable, then what is the value of $\Sigma\, p(X)$?

3. If X is a random variable and $p(x)$ is the probability of X, then what is $E(X)$?

4. A coin is tossed twice. What is the probability that both are heads?

5. A die is rolled once. What is the probability that the number is odd?

6. If c is a constant, then $E(c) = ?$

7. $E(X - E(X)) = ?$

8. What is a random experiment?

9. What is the variance of binomial distribution?

10. If $f(x)$ is a probability density function of normal distribution, then what is $\int_{-\infty}^{\infty} f(x)\,dx$?

11. What is the maximum value of the probability density function of a discrete probability distribution?

12. Which parameter controls the relative flatness of the normal distribution curve?

13. What is the mean deviation of normal distribution?

14. If $X \sim N(55, 49)$, then $\sigma = ?$

15. What is the sum of probabilities of a discrete distribution?

16. What is the probability density function of exponential distribution?

17. If $f(x, y)$ be the joint density function of (x, y) and $F_{xy}(x, y)$ be the cumulative distribution function of (x, y), then what is the relation between $f(x, y)$ and $F_{xy}(x, y)$?

18. If f_{xy} be the joint probability density function of (x, y) and $f_{xy}(x, y) = f_x(x)\, g_y(y)$, then what is the relation between x and y?

19. Define conditional distribution function.

20. If $F(x, y)$ be the joint distribution function of (x, y), then what is the value of $F(-\infty, +\infty)$?

21. If $E(X)$ be the expectation of X, then what is the value of $E[E(Y / X = x)]$?

22. If X and Y are two independent random variables, then find the value of $f_{Y/X}(y/x)$.

23. If X and Y be two random variables, then what is the regression curve of Y on X?

24. If X and Y are two independent random variables, then what is the value of $E(XY)$?

25. If r_{XY} be the correlation coefficient between X and Y, then what are the limits of r_{XY}?

26. If X and Y be two standard normal variates with correlation coefficient ρ, then what is the line of regression of Y on X?

27. If X and Y be two independent random variables, then define $M_{X,Y}(t_1, t_2)$.

28. Define positive correlation.

29. At which point, do the two lines of regression intersect each other?

30. Explain $b_{yx} = 105$, where b_{YX} is the regression coefficient of Y on X.

31. What is the relation between correlation coefficient and regression coefficient?

32. What are the limits of regression coefficients?

33. State Markov's inequality.

34. Under what condition, $x_n \xrightarrow{P} a$, where a is any constant?

35. If $\bar{x}_n$ be the arithmetic mean of n iid random variables and $V(x_i) = \sigma^2$, then what is the value of $V(\bar{x}_n)$?

36. State the necessary and sufficient conditions that a sequence of random variables follows the central limit theorem.

37. If the variables are uniformly bounded, then state the necessary and sufficient conditions that WLLN holds.

38. Under what condition, $x_n - \mu_n \xrightarrow{P} 0$?

39. State Lindeberg-lèvy central limit theorem.

40. If $x_n \to x$, $y_n \to y$, then under what condition $x_n + y_n \to x + y$?

SECTION—B

Answer any *five* of the following questions : 2×5=10

41. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number multiple of 3 or 5?

42. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

43. Find the mean of binomial distribution $B(n, p, q)$, where p, q and n are the probabilities of success and failure and number of trials respectively.

44. If m is the mean of Poisson distribution, then find the variance.

45. Define marginal probability mass function and marginal probability density function.

46. If A and B be two mutually exclusive events, then prove that

$$E[X|A \cup B] = \frac{P(A)E(X|A) + P(B)E(X|B)}{P(A \cup B)}$$

where

$$E(X|A) = \frac{1}{P(A)} \sum_{x_i \in A} x_i \, P(X = x_i)$$

47. If one of the regression coefficients is greater than 1, then prove that the other is less than 1.

48. Prove that correlation coefficient is independent of the change of origin and scale.

49. State Bernoulli's law of large numbers.

50. State weak law of large numbers.

SECTION—C

Answer *any* **five** of the following questions : 8×5=40

51. *(a)* What is the probability of the occurrence of a number that is odd or less than 5, when an unbiased die is rolled? 4

(b) A bag contains 4 blue, 2 red and 3 black balls. If 2 balls are drawn at random from the bag, and then another ball is drawn, what is the probability of getting 2 blue balls and 1 black ball? 4

52. *(a)* Define mathematical expectation of a random variable. If X and Y are two random variables, then show that

$$E(X+Y) = E(X) + E(Y)$$

and $E(XY) = E(X)E(Y)$ 4

(b) Determine the mean and variance of the random variable X having the probability distribution : 4

$X = x$	1	2	3	4	5	6	7	8	9	10
$P(x)$	0·15	0·10	0·10	0·01	0·08	0·01	0·05	0·02	0·28	0·20

53. *(a)* Find the mean and variance of binomial distribution. 4

(b) If X is binomially distributed with 6 trials and a probability of success equal to $\frac{1}{4}$ at each trial, then what is the probability of *(i)* exactly 4 successes and *(ii)* at least 1 success? 4

54. *(a)* Find the mean and variance of Poisson distribution. 4

(b) The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0·5. Find the probability that in a particular week there will be

(i) less than 2 accidents;

(ii) more than 2 accidents. 4

55. *(a)* If X and Y be two random variables and their probability distribution is

$X \backslash Y$	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

then find

(i) $P(X \le 1, Y > 2)$

(ii) $P(X \le 1)$

(iii) $P(Y \le 3)$

(iv) $P(X < 3, Y \ge 4)$ 4

(b) X and Y be two random variables, then prove that

$$V(X) = E[V(X|Y)] + V[E(X|Y)]$$ 4

56. *(a)* If the joint density function of X and Y is

$$f(x, y) = \begin{cases} \frac{1}{2} x e^{-y}, & 0 < x < 2, y > 0 \\ 0 \quad , & \text{otherwise} \end{cases}$$

then find the distribution of $X + Y$. 6

(b) If

$$f(x, y) = \begin{cases} 2 - x - y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0 \quad , & \text{otherwise} \end{cases}$$

then find the marginal probability density function of $X = x$. 2

57. *(a)* If X and Y be two random variables and a, b, c and d be constants such that $a \ne 0$, $c \ne 0$, then prove that

$$r(ax + b, cy + d) = \frac{ac}{|ac|} r(x, y)$$ 4

(b) Prove that two independent random variables are uncorrelated. Also, prove with an example that the converse may not be true. 4

58. *(a)* If $(X, Y) \sim BvN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then find the marginal probability of X and Y. 4

(b) Find the angle between two lines of regression. Also interpret the result for $r_{xy} = 0$, where r_{xy} is the correlation coefficient between x and y. 3+1=4

59. *(a)* State and prove Chebyshev's inequality. 4

(b) Let $X_1, X_2, \ldots$ be iid variables with probability mass function

$$f(x) = \frac{\alpha^x}{1-\alpha}, \; x = 0, 1, 2, \ldots; \; 0 < \alpha < 1$$

Show that X_i's follow WLLN. 4

60. *(a)* Examine if the law of large numbers holds for the sequence of independent random variables $\{x_n\}$ with the distribution of x_n given by

$$f_n(x) = \begin{cases} \frac{1}{|x|^3}, & |x| > 1 \\ 0, & \text{otherwise} \end{cases}$$

4

(b) State and prove De Moivre-Laplace central limit theorem. 4

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