2020/TDC(CBCS)/ODD/SEM/
MTMHCC–502T/332

TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

MATHEMATICS

(5th Semester)

Course No. : MTMHCC–502T

(Multivariate Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Investigate for continuity at (1, 2)

$$f(x, y) = \begin{cases} x^2 + 2y & , (x, y) \neq (1, 2) \\ 0 & , (x, y) = (1, 2) \end{cases}$$

(b) Calculate $f_x(0, 0)$

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & , \quad x \neq 0, y \neq 0 \\ 0 & , \quad x = 0 = y \end{cases}$$

(c) Let

$$f(x, y) = \frac{y - x}{y + x} \cdot \frac{1 + x}{1 + y}$$

Then show that

$$\lim_{x \to 0} \lim_{y \to 0} f(x, y) \neq \lim_{y \to 0} \lim_{x \to 0} f(x, y)$$

(d) Find the directional derivatives of a scalar point function f in the direction of coordinate axis.

(e) State a necessary condition for $f(x, y)$ to have an extreme value at (a, b).

(f) Give an example of a function $f(x, y)$ having an extreme value at $(0, 0)$ even though the partial derivative f_x and f_y do not exist at $(0, 0)$.

(g) Find the stationary points of the function $x^3 + y^3 - 12x - 3y + 20$.

(h) Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at $(0, 0)$.

(i) Evaluate $\iint (x^2 + y)\, dx\, dy$ over the rectangle $[0, 1; 0, 2]$.

(j) Find the value of

$$\iint_E e^{y/x} dS$$

if the domain E of integration is the triangle bounded by the straight lines $y = x$, $y = 0$ and $x = 1$.

(k) Evaluate

$$\iint_{x^2 + y^2 \leq a^2} (x^2 + y^2) dx\, dy$$

by changing to polar coordinate.

(l) Define divergence of a vector field.

(m) Let f be a bounded function of x, y, z on a parallelopiped $R = [a, b; c, d; g, h]$. Define triple integral of f over R.

(n) When is a three-dimensional domain called regular with respect to an axis?

(o) Compute the integral

$$\iiint_E xyz\, dx\, dy\, dz$$

(p) Evaluate the integral by passing over to cylindrical coordinate

$$\int_0^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_0^a dz$$

(q) Define the line integral of a function f along a curve C.

(r) Evaluate the line integral $\int_C (x^2 dx + xy dy)$ taken along the line segment from (1, 0) to (0, 1).

(s) Show that the area of a domain A (with contour C) regular with respect to both the axes $= \frac{1}{2}\int_C (x dy - y dx)$.

(t) State Stokes' theorem.

SECTION—B

Answer *any* **five** questions

2. (a) Discuss the continuity of the function $f(x, y)$ at (0, 0), when

$$f(x, y) = \begin{cases} 2xy\dfrac{x^2 - y^2}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$ 5

(b) If

$$f(x, y) = \begin{cases} xy\tan(y/x) & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Show that $xf_x + yf_y = 2f$. 5

3. (a) Find the maximum value of the directional derivatives of $\phi = x^2yz$ at the point (1, 4, 1). 5

(b) Show that the repeated limits exist at the origin and are equal but the simultaneous limit does not exist where

$$f(x, y) = \begin{cases} 1, & \text{if } xy \neq 0 \\ 0, & \text{if } xy = 0 \end{cases}$$ 5

4. (a) Find the maxima and minima of $x^2 + y^2 + z^2$, subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$. 5

(b) Show the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8\,abc}{3\sqrt{3}}$. 5

5. (a) Discuss the method to determine the stationary points by using Lagrange's method of undetermined multipliers. 5

(b) Apply Lagrange's method of undetermined multipliers to find the minima of $u = x^2 + y^2 + z^2$, when $xy + yz + zx = 3a^2$. 5

6\. (a) Evaluate $\iint (y-2x)\,dx\,dy$ over $R = [1, 2; 3, 5]$. 5

(b) Evaluate $\iint y\,dx\,dy$ over the part of the plane bounded by the lines $y = x$ and the parabola $y = 4x - x^2$. 5

7\. (a) Show that

$$\iint_E \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\,dxdy = ab\frac{\pi}{4}\left(\frac{\pi}{2} - 1\right)$$

where E is the region in the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

(b) Evaluate : 5

$$\int_0^{\pi}\int_0^{\pi} |\cos(x+y)|\,dx\,dy$$

8\. (a) Evaluate $\iiint x^2y^2z^2\,dx\,dy\,dz$ taken throughout the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$. 5

(b) Find the value of $\int_C (x + y^2)dx + (x^2 - y)dy$ taken in the clockwise sense along the closed curve C formed by $y^3 = x^2$ and the chord joining $(0, 0)$ and $(1, 1)$. 5

9\. (a) Find the volume cut from a sphere of radius a by a right circular cylinder with b as radius of the base and whose axis passes through the centre of the sphere. 5

(b) Show that

$$\iiint_E z^2\,dx\,dy\,dz$$

where E is the region of the hemisphere $z \geq 0,\ x^2 + y^2 + z^2 \leq a^2$ is $\frac{2}{15}\pi a^5$. 5

10\. (a) State Green's theorem in the plane. Verify the theorem for

$$\oint_C (xy + y^2)dx + x^2dy$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 1+4=5

(b) Use Stokes' theorem to find the line integral

$$\int_C x^2y^3dx + dy + zdz$$

where C is the circle $x^2 + y^2 = a^2,\ z = 0$. 5

11. *(a)* Evaluate the surface integral

$$I = \iint_S y^2 z dx\, dy + xz dy\, dz + x^2 y dz dx$$

where S is the outer side of the surface situated in the first octant and formed by the paraboloid of the revolution $z = x^2 + y^2$, cylinder $x^2 + y^2 = 1$ and the coordinate planes. 5

(b) If

$$\frac{\partial g}{\partial x} = \frac{\partial f}{\partial y}$$

for every point of E, and if A and B are two points of E, then prove that the line integral $\int f dx + g dy$ has the same value for every path from A to B, provided the path lies in E. 5

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