2021/TDC/CBCS/ODD/
MATDSC/GE–301T/327A

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(3rd Semester)

Course No. : MATDSC/GE–301T

(Real Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. Find the supremum of the set $\{1+(-1)^n \frac{1}{n} : n \in \mathbb{N}\}$.

2. Give an example of a countable collection of finite sets whose union is not finite.

3. State Archimedean property of $\mathbb{R}$.

4. Define bounded above set.

5. State completeness property of $\mathbb{R}$.

6. Can a finite set be closed? Justify.

7. Give an example of an open set which is not an interval.

8. Give an example of a set which is neither open nor closed.

9. Give an example of a set having no limit point.

10. Give an example of a set which is both open and closed.

11. Under what condition a monotone sequence is convergent?

12. Define Cauchy sequence.

13. Give example of divergent sequences $< x_n >$ and $< y_n >$ such that $< x_n + y_n >$ is convergent.

14. Define bounded sequence.

15. Give an example of a sequence which is unbounded but has a convergent subsequence.

16. State the necessary condition for the convergence of a series.

17. Define an absolutely convergent series.

18. State the conditions for convergent and divergent of the geometric series $\sum r^n$.

19. Give an example of an alternating series.

20. Under what condition

$$\sum \frac{1}{n(\log n)^p}$$

is divergent?

21. Let f, g be two functions defined as follows :

$$f(x) = \sqrt{x}, \ \forall\, x \geq 0, \ g(x) = x^2 + 1, \ \forall\, x \in \mathbb{R}$$

What is the domain of the function $f \circ g$?

22. Give an example of a function which is bounded in an interval but is not continuous at that interval.

23. Does $\lim\limits_{x \to 0} e^{\frac{1}{x}}$ exist?

24. Fill in the blank :

A function continuous in a closed interval is _____ therein.

25. Give an example of a function, where $|f|$ is continuous but f is not.

SECTION—B

Answer any *five* of the following questions : 2×5=10

26. Show that least upper bound of a bounded above set is unique.

27. If S is a bounded non-empty subset of $\mathbb{R}$, then prove that

$$\inf S \leq \sup S$$

28. Prove that the interior of a set S is a subset of S.

29. Show that union of two closed sets is a closed set.

30. Prove that $\{\frac{1}{n}\}$ is a Cauchy sequence.

31. Prove that

$$\lim_{n \to \infty} \frac{n}{n+1} = 1$$

32. State Leibnitz's theorem.

33. Define conditionally convergent series and give an example.

34. Show that the function f defined by $f(x) = x - |x|, \forall x \in \mathbb{R}$ is continuous at $x = 0$.

35. Show that the function defined as

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$$

has removable discontinuity at the origin.

SECTION—C

Answer any *five* of the following questions : 8×5=40

36. *(a)* Prove that the set of real numbers in [0, 1] is uncountable. 4

(b) If A and B are bounded subsets of real numbers, then prove that $A \cap B$ and $A \cup B$ are also bounded. 4

37. *(a)* Show that the set of integers $\mathbb{Z}$ is countable. 4

(b) Prove that the set $\mathbb{N}$ of natural numbers is not bounded above. 4

38. *(a)* Show that arbitrary union of open sets is open. Give example to show that the same is not true for closed sets. 4+2=6

(b) Prove that every open interval is an open set. 2

39. *(a)* Show that a set A is closed if and only if $A = \overline{A}$, where $\overline{A}$ is the closure of A. 4

(b) Let A, B be subsets of $\mathbb{R}$ and $D(A)$, $D(B)$ be the derived sets of A and B respectively. Show that

$$D(A \cup B) = D(A) \cup D(B)$$

Give example to show that $D(A \cap B)$ may not be equal to $D(A) \cap D(B)$. 4

40. *(a)* Let $\lim_{n \to \infty} x_n = x$ and $\lim_{n \to \infty} y_n = y$. Then prove that

$$\lim_{n \to \infty} (x_n + y_n) = x + y$$ 4

(b) Show that the sequence $< x_n >$, given by $x_1 = 1$, $x_{n+1} = \sqrt{2x_n}$, $n \in \mathbb{N}$ converges to 2. 4

41. *(a)* State and prove squeeze theorem. 4

(b) Prove that the sequence $< x_n >$ defined by

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}, n \in \mathbb{N}$$

is not a Cauchy sequence. 4

42. *(a)* Test for convergence the series

$$\frac{\alpha}{\beta} + \frac{(1+\alpha)}{(1+\beta)} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \cdots$$ 4

(b) Show that the series

$$x + \frac{x^2}{L^2} + \frac{x^3}{L^3} + \cdots$$

converges absolutely for all values of x. 4

43. *(a)* Test the convergence of the series

$$\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \cdots$$

for $x > 0$. 4

(b) Show that the series

$$\sum \frac{(-1)^{n+1}}{n^P}$$

is absolutely convergent for $P > 1$, but conditionally convergent for $0 < P \le 1$. 4

44. *(a)* Let f be a real-valued function defined on $I \subseteq \mathbb{R}$ and $c \in I$. Then prove that f is continuous at c if and only if for every sequence $\{x_n\}$ in I with $\lim_{n \to \infty} x_n = c$, we have

$$\lim_{n \to \infty} f(x_n) = f(c)$$ 4

(b) Prove that the image of a closed interval under a continuous function is a closed interval. 4

45. (a) If a function f is continuous on $[a, b]$ and $f(a) \neq f(b)$, then prove that it assumes every value between $f(a)$ and $f(b)$. 5

(b) Let f be a function on $\mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ -1, & \text{when } x \text{ is irrational} \end{cases}$$

Show that f is discontinuous at every point of $\mathbb{R}$. 3

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2021/TDC/CBCS/ODD/
MATSEC–301T (A/B/C)/328

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(3rd Semester)

Course No. : MATSEC–301T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates are to answer *either* Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MATSEC–301T (A)

(Logic and Sets)

SECTION—A

Answer any *fifteen* of the following questions : 1×15=15

1. Write the negation of the statement :

p : Every natural number is greater than 0

2. Identify the type of 'Or' used in the following statement and check whether the statement is True or False :

q : $\sqrt{7}$ is a rational number or an irrational number

3. Write down the contrapositive of the statement :

p : If $\frac{a}{b}$ and $\frac{b}{c}$ are integers, then $\frac{a}{c}$ is an integer.

4. Rewrite the following statement so that it is clear that it is an implication :

q : A differentiable function is continuous.

5. Rewrite each of the following with universal and existential quantifiers :

(a) Not all continuous functions are differentiable.

(b) There is no smallest integer.

6. Write the negation of each of the following :

(a) For every real number x, there is an integer n such that $n > x$.

(b) There exists an infinite set whose proper subsets are all finite.

7. If 0 denotes a contradiction, show that

$$p \wedge 0 \Leftrightarrow 0$$

8. If 1 denotes a tautology, show that $p \vee 1 \Leftrightarrow 1$.

9. Justify True or False :

$$A \subseteq B \Rightarrow A^C \subseteq B^C$$

10. What is $A \cap ((A \cap B)^C)$?

11. How many elements are in the power set of the power set of the empty set?

12. What is $\mathbb{N} \cap (-5, 5)$?

13. Justify True or False :

$$((A \setminus B) \subseteq (B \setminus A)) \rightarrow (A \subseteq B)$$

14. How many subsets of B of $\{1, 2, 3, \cdots, n\}$ have the property that $B \cap \{1, 2, 3\} = \varnothing$? Explain.

15. If $A = [-4, 4]$ and $B = [0, 5]$, then what is $A \setminus B$ and $B \setminus A$?

16. Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$, for any sets A, B and C.

17. Give example of a relation that is neither reflexive nor symmetric nor transitive.

18. Define a partial order relation on a non-empty set.

19. Is every reflexive relation an identity relation? Justify.

20. Define partition of a set.

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

22. Construct a truth table for the following compound statement :

$$p \rightarrow \sim (q \vee p)$$

23. Show that there is no largest integer.

24. Show that an implication and its contrapositive are logically equivalent.

25. If $A = \varnothing$, find $P(P(P(A)))$.

26. Show that

$$P(A \cap B) = P(A) \cap P(B)$$

27. Prove that

$$\left(\bigcup_{i=1}^{n} A_i\right)^C = \bigcap_{i=1}^{n} A_i^C$$

28. Show that the number of elements in the power set of a set having m elements is 2^m.

29. Determine the partition of $\mathbb{Z}$ produced by the relation 'congruence modulo 5'.

30. Prove that any finite (non-empty) poset must contain maximal and minimal elements.

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. Construct a truth table for the following compound statement : 5

$$(p \vee q) \leftrightarrow [((\sim p) \wedge r) \rightarrow (q \wedge r)]$$

32. *(a)* Fill in the blanks so that the resulting statement is equivalent to the implication $p \Rightarrow q$: 3

(i) _____ is necessary for _____

(ii) _____ only if _____

(iii) _____ is sufficient for _____

(b) Using the concept of contrapositive, prove that—

"If the average of four different integers is 10, then one of the integers is greater than 11." 2

33. *(a)* Let x be a real number. Show that the following are equivalent :

(i) $x = \pm 1$

(ii) $x^2 = 1$

(iii) If a is any real number, then $ax = \pm a$. 3

(b) Suppose m and n are integers such that $n^2 + 1 = 2m$. Prove that m is the sum of the squares of two integers. 2

34. Using algebra of propositions, establish the following logical equivalences : 3+2=5

(a) $p \to (q \to r) \Leftrightarrow (p \wedge \sim r) \to \sim q$

(b) $p \to (q \vee r) \Leftrightarrow (p \wedge \sim q) \to r$

35. *(a)* If A and B are non-empty sets, show that

$$A \times B = B \times A \text{ iff } A = B$$ 3

(b) Show that

$$A \times B \subseteq C \times D \Rightarrow A \subseteq C \text{ and } B \subseteq D$$ 2

36. *(a)* Show that for any sets A, B and C

$$(A \cap B) \times C = (A \times C) \cap (B \times C)$$ 3

(b) Let $n \geq 1$ be a natural number. How many elements are in the set

$$\{(a, b) \in \mathbb{N} \times \mathbb{N} / a \leq b \leq n\}?$$

Explain. 2

37. *(a)* Assume that $P(A) = P(B)$, show that $A = B$. 3

(b) Justify if the following is true : 2

$$P(A) \neq \varnothing \Leftrightarrow A \neq \varnothing$$

38. *(a)* Show that

$$\bigcap_{n=1}^{\infty} \left[0, \frac{1}{n}\right] = \{0\}$$ 3

(b) Justify True or False : 2

$$(A \cup B) \subseteq A \cap B \to A = B$$

39. *(a)* Prove that a poset has atmost one maximum element. 2

(b) Prove that a glb of two elements in a poset $(A, \leq)$ is unique whenever it exists. 3

40. *(a)* Show that any two equivalence classes are either disjoint or identical. 2

(b) For natural numbers x and y, define a relation R as $(x, y) \in R$ iff $x^2 + y$ is even. Show that R is an equivalence relation. 3

OPTION—B

Course No. : MATSEC–301T (B)

(Programming in C)

SECTION—A

Answer any *fifteen* of the following questions :
1×15=15

1. Write the syntax for declaring an integer variable x in C.

2. How will you write the arithmetic expression $a^2 + 5a - 7$ in C?

3. Write the general form of scanf statement.

4. Write the syntax of variable declaration.

5. What are relational operators?

6. Write the following as a C expression :
$x + y$ is less than 5

7. Write the C expression for
$$x = -b + \sqrt{b^2 - 4ac}$$

8. What are logical operators?

9. Write the general syntax of for loop.

10. What is the purpose of continue statement?

11. Write the general syntax of do-while loop.

12. Give example of an exit-controlled loop.

13. Write the general syntax of function prototype declaration.

14. When is a function defined of void type?

15. What is a recursive function?

16. Can a function have more than one return statement?

17. Write the general syntax for declaring an array.

18. Write the general syntax of initializing a one-dimensional array.

19. If x is an array of size 5, how are the elements of x listed?

20. What is a two-dimensional array?

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Write the rules for naming variables in C.

22. Write a program to display the words 'Hello World!' on screen.

23. Write a C program to find the area of a rectangle of given sides.

24. Determine the values of each of the following logical expressions, given that $a=5$, $b=10$, $c=-2$:

(a) $a>b \,||\, a>c$

(b) $a==c \,\&\&\, b>a$

25. Write a program to display the larger of two given numbers.

26. Explain entry controlled and exit controlled loop.

27. Write a simple program to compute the product of two numbers using a user-defined function.

28. Explain actual arguments and formal arguments with regards to functions in C.

29. Explain the process of initializing a two-dimensional array.

30. Write a note on the uses of arrays in programming.

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. (a) Describe the various types of constants in C. 3

(b) Explain the type definition feature in C. 2

32. Describe the data types in C. 5

33. (a) Write a C program to compute the sum of the squares of three given numbers. 3

(b) Write the rules for precedence of arithmetic operators. 2

34. Explain integer arithmetic, real arithmetic and mixed-mode arithmetic in C. Illustrate with suitable examples. 5

35. Write a C program to compute the sum of the squares of first n natural numbers. 5

36. Explain the use of switch statement with suitable example. 5

37. Write a brief note on user-defined functions, their types, general syntax, and advantages. Illustrate your answer with suitable examples. 5

38. Write a C program to compute the sum of first n natural numbers using function. 5

39. Write a program to find the sum of two one-dimensional arrays entered by the user. 5

40. Write a program to find the largest element in an integer array. 5

OPTION—C

Course No. : MATSEC–301T (C)

(Classical Algebra and Trigonometry)

SECTION—A

Answer any *fifteen* of the following questions : $1\times15=15$

1. If A is a skew-symmetric matrix of odd order, what is the determinant of A? Justify your answer.

2. Define nilpotent matrix.

3. If A is a 4×4 matrix with $|A|=5$, what is the determinant of the adjoint of A?

4. What can you say about the diagonal entries of a skew-Hermitian matrix?

5. What is the rank of the identity matrix of order 5?

6. Define Echelon form of a matrix.

7. If A is a 3×3 non-singular matrix, what is the rank of A^{-1}?

8. What is the condition that a system of linear equations $Ax = B$ is consistent?

9. Find the sum of roots of the equation

$$2x^4 - 5x^3 + x^2 - x + 2022 = 0$$

10. State Descartes' rule of signs.

11. Write the cubic equation, given two of its roots are 1 and $1+i$.

12. Find the equation whose roots are reciprocal of those of $2x^2 + 3x + 1 = 0$.

13. State DeMoivre's theorem.

14. If ω is an imaginary cube root of unity, evaluate $\omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6$.

15. Write the expansion of $\sin\theta$ in ascending powers of θ.

16. Find the value of $e^{i\pi/4}$.

17. Write Gregory's series.

18. Write the formula for the sum of the cosines of n angles in AP.

19. Show that

$$\cosh^2\theta - \sinh^2\theta = 1$$

20. Express $\sin(a+ib)$ in the form $x+iy$ where a, b, x and y are real.

SECTION—B

Answer any *five* of the following questions : 2×5=10

21. Show that the matrix

$$A = \begin{pmatrix} a+ic & -b+id \\ b+id & a-ic \end{pmatrix}$$

is unitary if and only if

$$a^2 + b^2 + c^2 + d^2 = 1$$

22. Find the adjoint of the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

23. Reduce the matrix to Echelon form

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix}$$

24. For what value of α does the system

$$\alpha x + y + 2z = 0$$
$$x + y - z = 0$$
$$2x + 3y = 0$$

has non-trivial solution?

25. Apply Descartes' rule of signs to discuss the nature of roots of the equation

$$x^4 + x^2 + x - 2 = 0$$

26. Solve the equation

$$x^3 - 5x^2 - 16x + 80 = 0$$

given that it has two roots whose sum is zero.

27. Find all possible values of $i^{1/5}$.

28. If $x + \frac{1}{x} = 2\cos\frac{\pi}{7}$, show that

$$x^7 + \frac{1}{x^7} = -2$$

29. Show that

$$\frac{\pi}{8} = \frac{1}{1\times 3} + \frac{1}{5\times 7} + \frac{1}{9\times 11} + \cdots$$

30. If $x + iy = \sin(a + ib)$, show that

$$\frac{x^2}{\cosh^2 b} + \frac{y^2}{\sinh^2 b} = 1$$

SECTION—C

Answer any *five* of the following questions : 5×5=25

31. Prove that every square matrix can be expressed uniquely as the sum of a symmetric and a skew-symmetric matrix. 5

32. *(a)* Show that the inverse of a matrix is unique, if it exists. 3

(b) Check if the matrix

$$A = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

is orthogonal. 2

33. Reduce to normal form

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -7 \end{pmatrix}$$

Hence find its rank. 4+1=5

34. Solve the system of linear equations : 5

$$x+2y+3z=11$$
$$x-2y+4z=3$$
$$x+2y-z=-1$$

35. If the equation $x^3+px^2+qx+r=0$ has roots α, β and γ, find $\sum\alpha^3$ and $\sum\alpha^2\beta$ in terms of p, q and r. 5

36. If α, β and γ are the roots of $x^3+qx+r=0$, then find the equation whose roots are

$$\frac{\beta+\gamma}{\alpha^2},\ \frac{\gamma+\alpha}{\beta^2} \text{ and } \frac{\alpha+\beta}{\gamma^2}$$ 5

37. Prove that

$$\frac{\sin^3\theta}{3!} = \frac{\theta^3}{3!} - \frac{(1+3^2)\theta^5}{\underline{|5}} + (1+3^2+3^4)\frac{\theta^7}{\underline{|7}} + \cdots$$ 5

38. If

$$(1+x)^n = p_0 + p_1x + p_2x^2 + \cdots$$

show that

(i) $p_0 - p_1 + p_4 - \cdots = 2^{n/2}\cos\frac{n\pi}{4}$

(ii) $p_0 - p_3 + p_4 - \cdots = 2^{n/2}\sin\frac{n\pi}{4}$ 5

39. Find the sum

$$\sqrt{1+\sin\alpha} + \sqrt{1+\sin 2\alpha} + \sqrt{1+\sin 3\alpha} + \cdots + \sqrt{1+\sin n\alpha}$$ 5

40. Prove that

$$\pi = 2\sqrt{3}\left[1 - \frac{1}{3^2} + \frac{1}{5\times 3^2} - \frac{1}{7\times 3^3} + \cdots\right]$$ 5

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