2019/TDC/ODD/SEM/MTMDSC/
MTMGE–301T/260

TDC (CBCS) Odd Semester Exam., 2019

MATHEMATICS

(3rd Semester)

Course No. : MTMDSC/MTMGE–301T

(Real Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *four* of the following : 1×4=4

(a) Define finite set and give an example.

(b) Find a lower bound of the set of positive real numbers.

(c) Give an example of a countable collection of finite sets whose union is not finite.

(d) Let

$$S = \left\{1 - \frac{(-1)^n}{n};\ n \in \mathbb{N}\right\}$$

Find sup S.

(e) Give an example of a set which is bounded below but not bounded above.

2. (a) Show that the greatest lower bound of a set bounded below is unique. 2

Or

(b) Show that the set of all odd natural numbers is countable.

3. (a) Prove that a countable union of countable sets is countable. 4

(b) State and prove Archimedean property of $\mathbb{R}$. 1+3=4

OR

4. (a) Prove that the set of rational numbers is not order complete. 5

(b) Show that the supremum of a nonempty set S of real numbers, whenever it exists, is unique. 3

UNIT—II

5. Answer any *four* of the following : 1×4=4

(a) What is the derived set of Q?

(b) Define limit point of a subset of $\mathbb{R}$.

(c) Give an example of a set which is neither closed nor open in $\mathbb{R}$.

(d) Obtain the derived set of the set

$$\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$$

(e) Give an example of an open set which is not an interval.

6. (a) Obtain the derived set of the following sets : 2

(i) {1, 2, 3, 4, 5, 6}

(ii) {1, 2, 3, 4, ⋯, 500}

Or

(b) Prove that the union of two open intervals is not necessarily an open interval.

7. (a) Prove that the intersection of any finite number of open sets is open. 4

(b) State and prove Bolzano-Weierstrass theorem. (for sets) 4

OR

8\. (a) Prove that a set is closed if its complement is open. 3

(b) If a sequence of closed intervals $[a_n, b_n]$ is such that each member $[a_{n+1}, b_{n+1}]$ is contained in the preceding one $[a_n, b_n]$ and $\lim(b_n - a_n) = 0$, then prove that there is one and only one point common to all the intervals of the sequence. 5

UNIT—III

9\. Answer any *four* of the following : 1×4=4

(a) Show that the sequence $\{a_n\}$, where $a_n = \frac{n+1}{n}$ is convergent.

(b) Show that the sequence $\{x_n\}$, where $x_n = n^2$ is monotonically increasing.

(c) Define bounded sequence with example.

(d) Give an example of two divergent sequences X and Y such that their product XY converge.

(e) Give an example of a bounded sequence that is not a Cauchy sequence.

10\. (a) Show that every bounded sequence may not be a convergent sequence. 2

Or

(b) Applying Squeeze theorem, show that $\underset{n\to\infty}{\text{Lt}} \frac{\sin n}{n} = 0$

11\. (a) Prove that every convergent sequence is bounded. 4

(b) Show that the sequence $\{S_n\}$, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ cannot converge. 4

OR

12\. (a) State and prove Squeeze theorem. 4

(b) Define monotone sequence. Show that the sequence

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}, \ \forall n \in \mathbb{N}$$

is convergent. 1+3=4

UNIT—IV

13\. Answer any *four* of the following : 1×4=4

(a) Give an example of a convergent series which is not absolutely convergent.

(b) Justify if the series $\Sigma \frac{1}{3^n}$ is convergent or divergent.

(c) Show that the series

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$$

is not convergent.

(d) Can you give an example of a convergent series Σx_n and a divergent series Σy_n such that $\Sigma(x_n + y_n)$ is convergent?

(e) Give an example of a conditional convergent series.

14. (a) Test the convergence of Σx_n, where $x_n = \frac{n}{n^2+1}$. 2

Or

(b) Prove or disprove the series Σu_n is convergent if $\underset{n \to \infty}{\text{Lt}}\, u_n = 0$

15. (a) Test the convergence of the following series : 2+3=5

(i) $\sum(\sqrt{n^4+1}-\sqrt{n^4-1})$

(ii) $\frac{x}{1\cdot 2}+\frac{x^2}{2\cdot 3}+\frac{x^3}{3\cdot 4}+\cdots$

(b) Show that the series

$$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

converges absolutely for all values of x. 3

OR

16. (a) If Σu_n is a positive term series such that

$$\underset{n \to \infty}{\text{Lt}} \frac{u_n + 1}{u_n} = l$$

then the series

(i) converges, if $l < 1$;

(ii) diverges, if $l > 1$. 5

(b) Test the convergence of the series

$$1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$ 3

UNIT—V

17. Answer any *four* of the following : 1×4=4

(a) Write the sequential criterion for limit of a function.

(b) Using sequential criterion, show that $\underset{x \to 0}{\text{Lt}} \frac{1}{x}$ does not exist.

(c) Give an example of a function which is not continuous at any point of $\mathbb{R}$.

(d) Give an example of a function $f:[0, 1] \to \mathbb{R}$ that is discontinuous at every point of [0, 1] but such that $|f|$ is continuous on [0, 1].

(e) Define bounded function with example.

18. *(a)* For what value of a the function

$$f(x) = \begin{cases} x+3, & x \geq 1 \\ ax^2+8, & x < 1 \end{cases}$$

is continuous on $\mathbb{R}$? 2

Or

(b) Show that

$$\underset{x \to 0}{\text{Lt}} \sin \frac{1}{x}$$

does not exist.

19. *(a)* Prove that a function f defined on an interval I is continuous at a point $c \in I$ if and only if for every sequence $\{c_n\}$ in I converging to c, we have $\underset{n \to \infty}{\text{Lt}} f(c_n) = f(c)$. 5

(b) Show that the function f defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$$

is not continuous at any point of $\mathbb{R}$. 3

OR

20. *(a)* If f, g be two functions continuous at a point c, then show that the functions $f+g$, f_g are also continuous at c. 2+2=4

(b) Prove that if a function is continuous in a closed and bounded interval, then it is bounded therein. 4

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2019/TDC/ODD/SEM/
MTMSEC–301T (I/II/III)/179

TDC (CBCS) Odd Semester Exam., 2019

MATHEMATICS

(3rd Semester)

Course No. : MTMSEC–301T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Honours students will answer *either* from Option—I *or* Option—II and Pass students will answer Option—III

OPTION—I

(For Honours Students)

Course No. : MTMSEC–301T (I)

(LOGIC AND SETS)

UNIT—I

1. Answer any *three* questions : 1×3=3

(a) Classify each of the following statements as true or false :

(i) $4 \neq 1+3$ and $7 < \sqrt{50}$.

(ii) 6 is odd $\Rightarrow$ 2 is even.

(b) Rewrite each of the following statements so that it is clear that each is an implication :

(i) The reciprocal of a positive number is positive.

(ii) The product of rational numbers is rational.

(c) Write the negation of each of the following :

(i) x is real number and $x^2+1=0$

(ii) $2>3$ or $5>7$

(d) Write the contrapositive of $a\times b=0 \Rightarrow a=0$ or $b=0$.

2. Answer any *one* question : 2

(a) Using truth table, show that $p\wedge q \Rightarrow p\vee q$ is a tautology.

(b) Using truth table, show that $(\sim p\wedge q)\wedge(p\vee \sim q)$ is a contradiction.

3. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Every perfect square is of the form $4q$ or $4q+1$. Use the contrapositive of this implication to show that 111111 is not a perfect square. 3

(b) Write the truth values of $p\Rightarrow q$ and $q\Rightarrow p$ in the same truth table. 2

(c) Fill in the blanks so that the resulting statement is equivalent to the implication $p\Rightarrow q$: 3

(i) If _____ then _____.

(ii) _____ is necessary for _____.

(iii) _____ is sufficient for _____.

(d) A student gets admission in a college if he scores at least 50% in Mathematics and at least 60% in Science. What can you conclude about the scores of the student if he fails to get admission? 2

UNIT—II

4. Answer any *three* questions : 1×3=3

(a) Rewrite each of the following with universal and existential quantifiers :

(i) For real x, 2^x is never negative.

(ii) There is no largest integer.

(b) Write the negation of each of the following :

(i) For every $\varepsilon>0$, there exists $x\in\mathbb{R}$ such that $x>1-\varepsilon$.

(ii) There exists a, b, c such that $a(bc)\neq(ab)c$.

(c) If 0 denotes a contradiction, show that $p \vee 0 \Leftrightarrow p$.

(d) If 1 denotes a tautology, show that $p \wedge 1 \Leftrightarrow p$.

5. Answer any *one* question : 2

(a) Show that an implication and its contrapositive are logically equivalent.

(b) Write the negation of $\forall \varepsilon > 0\ (\exists \delta > 0$ such that $|x-a| < \delta \Rightarrow |f(x)-f(a)| < \varepsilon)$.

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Establish De Morgan's laws for the negation of the statements $p \vee q$ and $p \wedge q$. 2+2=4

(b) Use De Morgan's laws to show that $(\sim (p \vee q)) \wedge q$ is a contradiction. 1

(c) Using algebra of propositions, establish the following logical equivalences : 2+2=4

(i) $p \to (q \to r) \Leftrightarrow (p \wedge \sim r) \to \sim q$

(ii) $p \to (q \vee r) \Leftrightarrow (p \wedge \sim q) \to r$

(d) A set $A \subseteq \mathbb{R}$ is said to be bounded above if $\exists\, m \in \mathbb{R}$ such that $x \le m\ \forall\, x \in \mathbb{R}$. Using quantifiers, describe when is a set not bounded above. 1

UNIT—III

7. Answer any *three* questions : 1×3=3

(a) Justify true or false :

$$A \cup B = A \cup C \Rightarrow B = C$$

(b) Show that $A \subseteq B \Rightarrow B^C \subseteq A^C$.

(c) What is $A \cap ((A \cup B)^C)$?

(d) What is $\mathbb{N} \cap [-7, 7]$?

8. Answer any *one* question : 2

(a) If A and B are non-empty sets, show that $A \times B = B \times A$ iff $A = B$.

(b) Show that

$$A \times B \subseteq C \times D \Rightarrow A \subseteq C \text{ and } B \subseteq D$$

9. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Let $A_n = \{a \in \mathbb{Z} \mid a \le n\}$. Find $A_n \cap A_m$, $A_n \cup A_m$ and A_n^c for any $n, m \in \mathbb{Z}$. 3

(b) Let A and B be any sets. Show that $A \cap B = A$ iff $A \subseteq B$. 2

(c) Construct a bijection from $\mathbb{N}$ to $\mathbb{Z}$. Justify that it is a bijection. 3

(d) Construct a bijection from $(0, \infty)$ to $\mathbb{R}$. Justify. 2

UNIT—IV

10. Answer any *three* questions : 1×3=3

(a) How many subsets B of $\{1, 2, 3, \cdots, n\}$ have the property that $B \cap \{1, 2\} = \phi$? Explain your answer.

(b) Define symmetric difference of two sets A and B.

(c) Show that for any two sets A and B, $A \subseteq B \Leftrightarrow P(A) \subseteq P(B)$.

(d) If A has n elements, how many elements are there in $P(P(A))$?

11. Answer any *one* question : 2

(a) Show that the number of elements in the power set of a set having n elements is 2^n.

(b) Justify whether $P(A \cup B) = P(A) \cup P(B)$.

12. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Let $\{A_i | i \in I\}$ be an arbitrary class of sets, where I is a non-empty index set. Show that—

(i) $\left(\bigcup_{i \in I} A_i\right)^c = \bigcap_{i \in I} A_i^c$;

(ii) $\left(\bigcap_{i \in I} A_i\right)^c = \bigcup_{i \in I} A_i^c$. 2+2=4

(b) Justify if the following is true : 1

$$P(A) \neq \phi \Leftrightarrow A \neq \phi$$

(c) Show that

$$\bigcap_{n=1}^{\infty} \left[0, \frac{1}{n}\right] = \{0\}$$ 3

(d) Express $\mathbb{R}$ as a countable union of open intervals. 2

UNIT—V

13. Answer any *three* questions : 1×3=3

(a) Define a partial order relation on a non-empty set.

(b) Give example of a relation that is symmetric, transitive but not reflexive.

(c) Define partition of a set.

(d) Give example of a reflexive relation that is not antisymmetric.

14. Answer any *one* question : 2

(a) Draw the Hasse diagram for the inclusion relation on the power set of $A = \{x, y, z\}$.

(b) Determine the partition of $\mathbb{Z}$ produced by the relation 'congruence modulo 4'.

15. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Show that any two equivalence classes are either disjoint or identical. 2

(b) For natural numbers a and b, define a relation R as $(a, b) \in R$ iff $a^2 + b$ is even. Show that R is an equivalence relation. 3

(c) Show that any partition of a non-empty set defines an equivalence relation on the set. 3

(d) For $a, b \in \mathbb{R} \setminus \{0\}$, define $a \sim b$ iff $\frac{a}{b} \in Q$. Then ~ is an equivalence relation. Find equivalence class of 1 and show that $\sqrt{3}$ and $\sqrt{12}$ have the same equivalence class. 2

OPTION—II

(For Honours Students)

Course No. : MTMSEC–301T (II)

(PROGRAMMING IN C)

UNIT—I

1. Answer any *two* questions : 2×2=4

(a) Write appropriate declaration for the following group of variables :

Integer variables	:	*p, q*
Floating-point variables	:	*x, y, z*
Character variables	:	*a, b, c*

(b) What restrictions must be satisfied by all of the data items represented by an array?

(c) What is the purpose of the scant function? How is it used within a C program?

2. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Name and describe the four basic data types in C. 3

(b) What is a character constant? How do character constants differ from numeric-type constants? 3

(c) What is an assignment statement? What is the relationship between an assignment statement and an expression statement? 3

(d) What are the keywords in C? What restrictions are applied to their use? 3

UNIT—II

3. Answer any *two* questions : 2×2=4

(a) Describe logical NOT operator. What is its purpose? How many operands does it require?

(*b*) What is an expression? What are its components? Give example.

(*c*) Describe two equality operators included in C. How do they differ from the relational operator?

4. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) What is meant by associativity of an operator? What is the associativity of the arithmetic operator? Explain with the help of examples. 3

(*b*) Describe two different ways to utilize the increment and decrement operators. How do the two methods differ? 3

(*c*) What are unary operators? How many operands are associated with a unary operator? Give example. 3

(*d*) (*i*) How can multiple assignments be written in C?

(*ii*) In what general category do the # define and # include statement fall?

(*iii*) When should parentheses be included within an expression? 3

UNIT—III

5. Answer any *two* questions : 2×2=4

(*a*) What is meant by looping? Describe two different forms of looping.

(*b*) Describe the two different forms of the if-else statement. How do they differ?

(*c*) Write a loop to calculate the sum $2+5+8+11+\cdots+98$ using a for loop.

6. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) What is the purpose of the switch statement? Explain with the help of one example. 3

(*b*) Summarize the syntactic rules associated with the while statement. 3

(*c*) Can any of the three initial expressions in the 'for' statement be omitted? If so, what are the consequences of each omission? 3

(*d*) (*i*) What is the purpose of break statement?

(*ii*) Explain what happens when the following statement is executed :

if $(\text{abs}(x) < x\,\text{min})\; x = (x > 0)?\; x\,\text{min}{:}-x\,\text{min}$

(*iii*) Give one example of continue statement. 3

UNIT—IV

7. Answer any *two* questions : 2×2=4

(a) What is a function? State three advantages to the use of functions.

(b) What is the purpose of the keyword void? Where is the keyword used?

(c) What is recursion? What advantage is there in its use?

8. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) What are formal arguments? What are actual arguments? Give example. 3

(b) What are function prototypes? What is their purpose? 3

(c) What are differences between passing an array to a function and passing a single-valued data item to a function? 3

(d) *(i)* What is the purpose of the return statement?

(ii) Explain the meaning of each of the following prototypes :

int f (inta); and char f(void);

(iii) Following is the first line of a function definition. Explain.

float f(float a, float b) 3

UNIT—V

9. Answer any *two* questions : 2×2=4

(a) In what way does an array differ from an ordinary variable? Explain with example.

(b) What are subscripts? How are they written?

(c) *(i)* What value is automatically assigned to those array elements that are not explicitly initialized?

(ii) Describe the array that is defined in the following statement :

```
int p[2][4] = {
                {1, 3, 5, 7},
                {2, 4, 6, 8}
              };
```

10. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) State the rule that determines the order in which initial values are assigned to multidimentional array elements. 3

(b) What advantages are there in defining an array size in terms of a symbolic constant rather than a fixed integer quantity? 3

(c) How can a list of strings be stored within a two-dimentional array? What library functions are available to simplify string processing? 3

(d) When are array declaration required in a C program? How do such declaration differ from an array definition? 3

OPTION—III

(For Pass Students)

Course No. : MTMSEC-301T (III)

(CLASSICAL ALGEBRA AND TRIGONOMETRY)

UNIT—I

1. Answer any *three* questions : 1×3=3

(a) Define nilpotent matrix.

(b) Give an example of skew-symmetric matrix.

(c) State Jacobi's theorem.

(d) Define Hermitian matrix.

2. Answer any *one* question : 2

(a) Prove that orthogonal matrices are unimodular.

(b) Prove that determinant of skew-symmetric matrix of odd order is zero.

Answer any *one* question :

3. Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -4 & 1 \\ 3 & 4 & -3 \end{pmatrix}$$ 5

4. (a) Prove that $(AB)^T = B^T A^T$ assuming conformability for multiplication. 3

(b) Prove that inverse of a square matrix, if it exists, is unique. 2

UNIT—II

5. Answer any *three* questions : 1×3=3

(a) Define rank of a matrix.

(b) Under what condition the rank of the matrix $\begin{pmatrix} 2 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 0 & x \end{pmatrix}$ is 3?

(c) Define elementary matrix.

(d) When a matrix is said to be in echelon form?

6. Answer any *one* question : 2

(a) What do you mean by elementary transformations of a matrix?

(b) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \\ 3 & 2 & -1 \end{pmatrix}$$

Answer any *one* question :

7. Solve the following system of linear equations by Gaussian elimination method : 5

$$x - y + z = 10$$
$$x + 2y - z = 7$$
$$x + y - z = 8$$

8. Find the rank of

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & -3 & 0 & -7 \end{pmatrix}$$

by reducing it to normal form. 5

UNIT—III

9. Answer any *three* questions : 1×3=3

(a) State Descarte's rule of signs.

(b) Define reciprocal equation.

(c) What is the product of all roots of the equation $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n = 0$?

(d) Find the equation whose roots are reciprocal of the equation

$$3x^2 + 2x + 1 = 0.$$

10. Answer any *one* question : 2

(a) Find the values of p and q if all roots of the equation $x^3 + px^2 + qx + 8 = 0$.

(b) Form the equation with integral coefficients whose roots are $1, -\frac{1}{2}$ and 5.

Answer any *one* question :

11. If α, β, γ are the roots of the equation $x^3 - ax^2 + bx - c = 0$, then find the equation whose roots are $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$. 5

12. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the values of $\Sigma\alpha^3\beta^2$. 5

UNIT—IV

13. Answer any *three* questions : 1×3=3

(a) Write down the expansion of $\cos n\theta$.

(b) Write down the exponential values of $\tan x$.

(c) Write $\cos\alpha$ as series of ascending processes of α.

(d) State De Moivre's theorem.

14. Answer any *one* question : 2

(a) If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, then prove that $x_1, x_2 \cdots \infty = -1$.

(b) If $x = \frac{2}{\lfloor 1} - \frac{4}{\lfloor 3} + \frac{6}{\lfloor 5} - \frac{8}{\lfloor 7} + \cdots \infty$ and $y = 1 + \frac{2}{\lfloor 1} - \frac{2^3}{\lfloor 3} + \frac{2^5}{\lfloor 5} - \cdots \infty$, then show that $x^2 = y$.

Answer any *one* question :

15. *(a)* Show that sum of n, nth roots of units is 0. 2

(b) Prove that $\sin\alpha = \alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - \cdots \infty$. 3

16. *(a)* Separate real and imaginary parts of $\log(\alpha + i\beta)$. 2

(b) If

$$\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0,$$

then prove that $\Sigma\cos^2\alpha = \frac{3}{2}$. 3

UNIT—V

17. Answer any *three* questions : 1×3=3

(a) Write down Gregory's series.

(b) Define $\cosh x$.

(c) Prove that $\cosh^2 x - \sinh^2 x = 1$.

(d) Write down the sum of

$$\cos\alpha + \cos(\alpha+\beta) + \cdots + \cos(\alpha + \overline{n-1}\beta)$$

18. Answer any *one* question : 2

(a) Separate real and imaginary parts (x and y being real) of $\sinh(x + iy)$.

(b) Sum to n-terms of the series $\cos^2\alpha + \cos^2 3\alpha + \cos^2 5\alpha + \cdots$.

Answer any *one* question :

19. If $\sin x = n\sin(\alpha + x)$, $-1 < n < 1$, then expand x in a series of ascending processes of n. 5

20. Separate into real and imaginary parts of $\tan^{-1}(x + iy)$. 5

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