2021/TDC/CBCS/ODD/
MATHCC–101T/324

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(1st Semester)

Course No. : MATHCC–101T

(Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. If $y = \log x$, then find y_{99}.

2. Find by Leibnitz's formula, the *n*th derivative of $y = x^3 \sin x$.

3. If $\log y = \tan^{-1} x$, then prove that

$$(1 + x^2)y_2 + (2x - 1)y_1 = 0$$

4. State L'Hospital's rule.

5. Define rectangular and oblique asymptotes.

6. Find the vertical and horizontal asymptotes of the graph of

$$y = \frac{3x+1}{x^2-4}$$

7. Write the reduction formula for

$$\int_0^{\pi/2} \cos^n x\, dx$$

8. Evaluate :

$$\int_0^{\pi/2} \sin^6 x\, dx$$

9. Obtain the reduction formula for

$$\int \tan^n x\, dx$$

10. Write the Cartesian and parametric equation of a circle with radius r and centre at (a, b).

11. Let $x = \phi(t)$, $y = \psi(t)$ be the parametric equations of the curve AB, t being the parameter. Write down the formula for the arc length of AB.

12. Find the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$.

13. Show that $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ iff $\vec{a}$ and $\vec{c}$ are parallel.

14. Prove that

$$[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$$

15. If $\vec{u}(t)$ and $\vec{v}(t)$ be two differential functions of the scalar t, then show that

$$\frac{d(\vec{u} \times \vec{v})}{dt} = \vec{u} \times \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \times \vec{v}$$

SECTION—B

Answer any *ten* of the following questions : 3×10=30

16. If $y = x^{2n}$, where n is positive integer, then show that—

(a) $y_n = 2^n \{1.3.5.\cdots(2n-1)\} x^n$;

(b) $y_n = \frac{\lfloor 2n}{\lfloor n} x^n$.

17. Let

$$P_n = \frac{d^n}{dx^n}(x^n \log x)$$

Prove the relation

$$P_n = n \cdot P_{n-1} + \underline{|n-1}$$

Hence show that

$$P_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

18. If $y = \sin(m \sin^{-1} x)$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

Also find $y_n(0)$.

19. Evaluate :

$$\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$$

20. Find the asymptote of the curve

$$x^3 + y^3 = 3axy$$

21. Find the point of inflection, if any, of the curve

$$y = \frac{x^3}{a^2 + x^2}$$

22. Find the value of

$$\int_0^{\pi/4} \tan^6 x \, dx$$

23. Obtain the reduction formula for

$$\int \sec^n x \, dx$$

n being a positive integer greater than 1.

24. Obtain the reduction formula for

$$\int \sin^m x \cos^n x \, dx$$

where m, n are positive integers > 1.

25. Find the whole length of the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

26. Find the length of one complete arc of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

27. Find the area of the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by the latus rectum about x-axis.

28. Prove that

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

29. Find the parametric and non-parametric equations of the plane passing through three non-collinear points whose position vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$.

30. Show that the vector equation of the sphere on the join of two given points $\vec{a}$ and $\vec{b}$ as diameter is

$$(\vec{r}-\vec{a})\cdot(\vec{r}-\vec{b})=0$$

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MATHCC–102T/323

TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022

MATHEMATICS

(1st Semester)

Course No. : MATHCC–102T

(Higher Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* questions : 2×10=20

1. Find all the values of $1^{1/3}$.

2. Expand $\cos^2\theta$ in powers of θ.

3. Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$.

4. Give an example of the relation on the set of positive integers which is symmetric, transitive but not reflexive.

5. If $f : A \to B$ be bijective, then prove that $f^{-1} : B \to A$ is also bijective.

6. Prove that the map $f : Q \to Q$ defined by $f(x) = 3x + 2$ is invertible, where Q is the set of rational numbers.

7. State division algorithm.

8. Find gcd (256, 1166).

9. Find the remainder when 11^{35} is divided by 13.

10. Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4 + 16x^3 + 7x - 11 = 0$.

11. Find the condition that the equation $x^3 + px^2 + qx + r = 0$ may have two roots equal but of opposite signs.

12. If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, then form the equation whose roots are $\beta\gamma + \frac{1}{\alpha}$, $\gamma\alpha + \frac{1}{\beta}$ and $\alpha\beta + \frac{1}{\gamma}$.

13. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$$

14. Investigate for what values of λ and μ the equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have a unique solution.

15. Is the system of equations $x + y + z = 4$, $2x + 5y - 2z = 3$, $x + 7y - 7z = 5$ solvable?

SECTION—B

Answer any *five* questions : 10×5=50

16. *(a)* If

$$Z_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$$

then prove that $Z_1 Z_2 Z_3 \ldots \infty = -1$. 5

(b) Show that

$$\log\log(x + iy) = \frac{1}{2}\log(p^2 + q^2) + i\tan^{-1}\frac{q}{p}$$

where

$$p = \log\sqrt{x^2 + y^2} \text{ and } q = \tan^{-1}\frac{y}{x}$$ 5

17. *(a)* Prove that

$$\sin^2\theta\cos\theta = \theta^2 - \frac{5}{6}\theta^4 + \dots + (-1)^{n+1}\frac{3^{2n}-1}{4\,\lfloor 2n}\theta^{2n} + \dots$$ 5

(b) If $\sin x = n\sin(\alpha + x)$, $-1 < n < 1$, then expand x in a series of ascending powers of n. 5

18. *(a)* Let A be a non-empty set and R be an equivalence relation defined on A. Let $a, b \in A$ be two arbitrary elements, then prove that—

(i) $b \in [a] \Rightarrow [b] = [a]$;

(ii) either $[a] = [b]$ or $[a] \cap [b] = \phi$. 2+3=5

(b) If $f : A \to B$ and $g : B \to C$ be two mappings and $g \circ f$ be one-one and onto, then prove that f is one-one and g is onto. 5

19. *(a)* Let R and R' be two equivalence relations on a set A. Prove that—

(i) $R \cap R'$ is an equivalence relation in A;

(ii) $R \cup R'$ is not necessarily an equivalence relation in A. 3+2=5

(b) If $f : X \to Y$ be a mapping and A, B be subsets of X, then show that

$$f(A \cap B) \subseteq f(A) \cap f(B)$$ 5

20. *(a)* Prove that the well-ordering principle is equivalent to the principle of finite induction. 5

(b) Prove that no integer in the following sequences is a perfect square : 2½+2½=5

(i) 11, 111, 1111, 11111, ...

(ii) 99, 999, 9999, 99999, ...

21. *(a)* Show that the rank of the transpose of a matrix is the same as that of the original matrix. 5

(b) Prove that for each $n \geq 1$

$$\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$$

is an integer. 5

22. *(a)* Solve the equation

$$x^3 - 3x^2 + 12x + 16 = 0$$

by Cardan's method. 5

(b) If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4 - 4x + 3 = 0$, then find the values of $\Sigma\alpha^4$ and $\Sigma\frac{1}{\alpha^2\beta\gamma}$. 2+3=5

23. *(a)* Solve the reciprocal equation

$$x^4 + 10x^3 + 26x^2 + 10x + 1 = 0$$ 5

(b) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are—

(i) $\beta\gamma, \gamma\alpha, \alpha\beta$;

(ii) $\beta+\gamma, \gamma+\alpha, \alpha+\beta$. 2+3=5

24. *(a)* For what values of K, the following system of linear equations has unique solution, infinitely many solutions or no solution? 6

$$x + 2y + 3z = 1$$
$$x + 3y + 5z = 4$$
$$x + 2y + Kz = 5$$

(b) Show that the vectors $\alpha_1 = (1, 1, 0)$, $\alpha_2 = (1, 3, 5)$, $\alpha_3 = (2, 2, 0)$ in $\mathbb{R}^3$ are linearly dependent. 4

25. *(a)* Solve the following system of equations by Gaussian elimination method : 5

$$x + 2y - z = 6$$
$$3x - y - 2z = 3$$
$$4x + 3y + z = 9$$

(b) Show that the vectors

$$\alpha_1 = (1, 1, 0, 0), \quad \alpha_2 = (0, 0, 1, 1)$$
$$\alpha_3 = (1, 0, 0, 4), \quad \alpha_4 = (0, 0, 0, 2)$$

are linearly independent. 5

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