TDC (CBCS) Odd Semester Exam., 2020
held in March, 2021

MATHEMATICS

(1st Semester)

Course No. : MTMHCC–101T

(Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

1. Answer any *ten* of the following questions : 2×10=20

(a) Find y_n, if $y = \sin^2 x$.

(b) If $y = x^{n-1} \log x$, then prove that

$$y_n = \frac{(n-1)!}{x}$$

(c) State Leibnitz rule on successive differentiation.

(d) If $y = (x + \sqrt{1+x^2})^m$, then prove that

$$(1+x^2)y_2 + xy_1 - m^2 y = 0$$

(e) Evaluate :

$$\underset{x \to 0}{\text{Lt}} \frac{\sec x - 1}{x \sec x}$$

(f) Find the value of

$$\underset{x \to \frac{\pi}{2}}{\text{Lt}} \left(\frac{\pi}{2} - x\right) \tan x$$

(g) Find the asymptote parallel to coordinate axes of the curve

$$4x^2 + 9y^2 = x^2 y^2$$

(h) Show that the curve $y = x^3$ has a point of inflection at $x = 0$.

(i) Obtain a reduction formula for

$$\int \sin^n x \, dx$$

(j) If

$$I_n = \int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \left(\frac{n-2}{n-1}\right) I_{n-2}$$

then find the value of $\int \sec^7 x \, dx$.

(k) If

$$I_n = \int_0^{\pi/4} \tan^n x \, dx = \frac{1}{n-1} - I_{n-2}$$

then evaluate

$$\int_0^{\pi/4} \tan^8 x \, dx$$

(l) Find the value of $\int_0^{\pi/2} \cos^{10} x \, dx$.

(m) Find by integration the length of $y = 5x$ from $x = 0$ to $x = 5$.

(n) Find the length of the perimeter of the circle $x^2 + y^2 = a^2$.

(o) Show that the length of the arc of the curve $y = \log \sec x$ between $x = 0$ and $x = \frac{\pi}{6}$ is $\frac{1}{2} \log 3$.

(p) The circle $x^2 + y^2 = a^2$ revolves round the X-axis. Find the surface generated.

(q) Find the value of p so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar.

(r) Prove that $\vec{a} \times (\vec{b} \times \vec{a}) = (\vec{a} \times \vec{b}) \times \vec{a}$.

(s) Find the vector equation of a sphere whose centre is $\vec{c} = 2\hat{i} - \hat{j} + \hat{k}$ and radius 5 units.

(t) If $\vec{r} = (\cos nt)\hat{i} + (\sin nt)\hat{j}$, where n is a constant and t varies, then show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = n\hat{k}$$

SECTION—B

Answer *any* **five** questions

2. (a) If $ax^2 + 2hxy + by^2 = 1$, then show that

$$y_2 = \frac{h^2 - ab}{(hx + by)^3} \quad 3$$

(b) If $f(x) = \tan x$, then prove that

$$f^n(0) - {}^nC_2 f^{n-2}(0) + {}^nC_4 f^{n-4}(0) - \dots = \sin\left(\frac{n\pi}{2}\right)^4 \quad 3$$

3. (a) If $y^3 + 3ax^2 + x^3 = 0$, then show that

$$y^5 y_2 + 2a^2x^2 = 0 \quad 3$$

(b) If $y = (\sin^{-1} x)^2$, then show that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0 \quad 3$$

4. (a) If

$$\underset{x \to 0}{\text{Lt}} \frac{\sin 2x + a \sin x}{x^3}$$

be finite, then find the value of a and the limit. 3

(b) Evaluate : 3

$$\underset{x \to e}{\text{Lt}} (\log x)^{\frac{1}{1 - \log x}}$$

5. (a) Obtain the asymptote of the curve

$$x^3 + y^3 = 6x^2 \quad 3$$

(b) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Find also its point of inflection, if any. 3

6. (a) If $I_{m,n} = \int \sin^m x \cos^n x \, dx$, then prove that

$$I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \left(\frac{n-1}{m+n}\right) I_{m,n-2} \quad 3$$

(b) If $I_n = \int_0^{\pi/2} \sin^n x \, dx$, then show that

$$I_n = \left(\frac{n-1}{n}\right) I_{n-2}$$

Hence or otherwise find $\int_0^{\pi/2} \sin^6 x \, dx$. 3

7. *(a)* Obtain a reduction formula for

$$\int \cos^m x \cos nx \, dx$$

connecting with $I_{m-1,\, n-1}$. Hence find the value of

$$\int \cos^2 x \cos 3x \, dx$$ 3

(b) If

$$I_n = \int_0^{\pi/4} \tan^n x \, dx$$

then show that $n(I_{n-1} + I_{n+1}) = 1$. 3

8. *(a)* Find the whole length of the loop of the curve $3ay^2 = x(x-a)^2$. 3

(b) Show that the surface area of the solid generated by revolving the arc of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ from $\theta = 0$ to $\theta = \pi/2$ about X-axis is $\frac{6}{5}\pi a^2$. 3

9. *(a)* Show that the upper half of the curve $r = a(1-\cos\theta)$ is bisected by $\theta = \frac{2\pi}{3}$. Show also that the perimeter of the curve is $8a$. 3

(b) Find the surface of the solid formed by revolving the curve $r = a(1+\cos\theta)$ about the initial line. 3

10. *(a)* Show that the necessary and sufficient conditions that the three non-zero, non-collinear vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar in their scalar triple product must vanish. 3

(b) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\lambda = [\vec{a}\ \vec{b}\ \vec{c}]$, then show that for any vector $\vec{d}$

$$\lambda\vec{d} = (\vec{c}\cdot\vec{d})(\vec{a}\times\vec{b}) + (\vec{a}\cdot\vec{d})(\vec{b}\times\vec{c}) + (\vec{b}\cdot\vec{d})(\vec{c}\times\vec{a})$$

given that

$$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$$ 3

11. *(a)* Find the vector equation of a plane passing through two given points and parallel to a given vector. 3

(b) Prove that the necessary and sufficient conditions for a vector $\vec{r} = \vec{f}(t)$ to have a constant magnitude is

$$\vec{f}\cdot\frac{d\vec{f}}{dt} = 0$$ 3

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