TDC (CBCS) Odd Semester Exam., 2019

MATHEMATICS

(1st Semester)

Course No. : MTMGE/MTMDSC–101T

(Differential Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *four* of the following : 1×4=4

(a) Write the value of

$$\underset{x \to a}{\text{Lt}} \frac{x^n - a^n}{x - a}$$

where n is rational and $a > 0$.

(b) If

$$f(x) = x^2 \quad \text{for } x \geq 1$$
$$= x^2 + 2 \quad \text{for } x < 1$$

Does $\underset{x \to 1}{\text{Lt}} f(x)$ exist?

(c) Find $\underset{x \to 0}{\text{Lt}}\, x \sin \frac{1}{x}$.

(d) If

$$\underset{x \to a}{\text{Lt}} f(x) = l$$

what is the value of $\underset{x \to a}{\text{Lt}} \{k f(x)\}$, k being a constant?

(e) Find $\underset{x \to 0}{\text{Lt}} \frac{\tan x}{x}$.

2. (a) State Cauchy's criterion for the existence of limit of a function. 2

Or

(b) Evaluate

$$\underset{x \to 0}{\text{Lt}} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$$ 2

Answer Question No. **3** *or* Question No. **4** :

3. (a) Using ε-δ definition of limit, evaluate $\underset{x \to 0}{\text{Lt}}\, x^2 \cos \frac{1}{x}$. 3

(b) Examine if $\underset{x \to 0}{\text{Lt}} \frac{e^{1/x}}{1 + e^{1/x}}$ exists or not. 3

(c) Find $\underset{n \to \infty}{\text{Lt}} \frac{x^n - 1}{x^n + 1}$. 2

4. (a) Using Cauchy's criterion, show that $\underset{x \to 0}{\text{Lt}} \sin \frac{1}{x}$ does not exist. 4

(b) Evaluate : 2+2=4

(i) $\underset{n \to \infty}{\text{Lt}} \frac{x^n}{1 + x^n}$

(ii) $\underset{x \to 0}{\text{Lt}} \frac{a - \sqrt{a^2 - x^2}}{x^2}$

UNIT—II

5. Answer any *four* of the following : 1×4=4

(a) Is the function

$$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n$$

continuous for all real values of x?

(b) When is the function $f(x) = \frac{1}{x-a}$ discontinuous?

(c) Is $\underset{x \to a}{\text{Lt}} f(x) = f(a)$ always?

(d) Find

$$\frac{d}{dx}\{\log(\sec x + \tan x)\}$$

(e) Evaluate

$$\underset{h \to 0}{\text{Lt}} \frac{f(x+h) - f(x)}{h}$$

if $f(x) = x + 5$.

6. (a) Show that if the function $f(x)$ is differentiable at $x = a$ then it is also continuous at $x = a$. 2

Or

(b) Examine the continuity of $f(x)$ at $x = 0$ if

$$f(x) = \begin{cases} 3 + 2x, & -\frac{3}{2} < x \le 0 \\ 3 - 2x, & 0 < x < \frac{3}{2} \end{cases}$$ 2

Answer Question No. 7 *or* Question No. 8 :

7. (a) Prove that if $f(x)$ be continuous at $x = a$ and $f(a) \neq 0$, then in the neighbourhood of $x = a$, $f(x)$ has the same sign as that of $f(a)$. 4

(b) Show that $f(x) = |x - 1|$ is not differentiable at $x = 1$, though it is continuous there. 4

8. (a) A function f is defined as

$$f(x) = x^2 \sin\frac{1}{x}, \quad x \neq 0$$
$$= 0, \quad x = 0$$

show that $f(x)$ is continuous at $x = 0$. Also examine the differentiability of the function at $x = 0$. 4

(b) $f(x) = x$ for $0 < x < 1$
$= 2 - x$ for $1 \le x \le 2$
$= x - \frac{1}{2}x^2$ for $x > 2$

Examine the continuity of $f(x)$ at $x = 1$, $x = 2$. 2+2=4

UNIT—III

9. Answer any *four* of the following : 1×4=4

(a) Find y_n if $y = \sin 3x$.

(b) Evaluate y_3 if $y = \log(x + a)$.

(c) Using Leibnitz's theorem, differentiate n times the following :

$$(1 - x^2)y_2 - xy_1 - 2 = 0$$

(d) Define homogeneous function.

(e) What is the degree of the function $f(x, y) = ax^2 + 2hxy + by^2$?

10. (a) State Leibnitz's theorem on successive differentiation. 2

Or

(b) State Euler's theorem on homogeneous function of degree n in two variables x and y. 2

Answer Question No. **11** *or* Question No. **12** :

11. (a) If $y = x^{2n}$, where n is a positive integer, show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5, \cdots (2n-1)\} x^n$$ 3

(b) (i) Find f_{xx} if $f(x, y) = e^{x^2 + xy + y^2}$. 2

(ii) If $u = \log(x^2 + y^2)$, prove that,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ 3

12. (a) If $y = \sin^{-1} x$, show that

(i) $(1 - x^2)y_2 - xy_1 = 0$

(ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$

2+3=5

(b) If

$$u = \tan^{-1}\left(\frac{x^3 - y^3}{x - y}\right)$$

prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$ 3

UNIT—IV

13. Answer any *four* of the following : 1×4=4

(a) Find the slope of the tangent to the curve $y = x^2$ at (1, 1).

(b) Write the equation of the normal to the curve $f(x, y)$ at the point (x, y).

(c) Write down the expressions for subtangent and subnormal for a plane curve in Cartesian form.

(d) What is the condition of orthogonality of two curves $r = f(\theta)$ and $r = Q(\theta)$?

(e) Write down the expression for radius of curvature when the equation of the curve is in intrinsic form.

14. *(a)* Find the polar subtangent for the curve $\frac{l}{r} = 1 + e\cos\theta$. 2

Or

(b) Find the radius of curvature at the point (s, ψ) of the curve

$$s = a\sec\psi\tan\psi + a\log(\sec\psi + \tan\psi)$$ 2

Answer Question No. **15** *or* Question No. **16** :

15. *(a)* Show that the portion of the tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted between the axis is of constant length. 4

(b) Find the radius of curvature for the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = 0$. 4

16. *(a)* Prove that the subnormal at any point of a parabola is of constant length and the subtangent varies as the abscissa of the point of contact. 2+2=4

(b) Find the radius of curvature at any point (s, ψ) on the curve $s = 4a\sin\psi$. Also show, for any curve

$$\frac{1}{\rho^2} = \left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2$$ 1+3=4

UNIT—V

17. Answer any *four* of the following : 1×4=4

(a) What is the necessary condition for existence of maximum or minimum of a function $y = f(x)$?

(b) Evaluate

$$\underset{x \to 0}{\mathrm{Lt}} \frac{x - \sin x}{x^2}$$

(c) State Cauchy's mean value theorem.

(d) Write Taylor's series in finite form.

(e) Which of the following is correct?

(i) $\cos x = x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$

(ii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

18. Evaluate $\underset{x \to 0+0}{\mathrm{Lt}} (\sin x \log x)$. 2

Or

Give the geometrical interpretation of Rolle's theorem. 2

Answer Question No. **19** *or* Question No. **20** :

19. (a) Show that the maximum values of $x^{1/x}$ is $e^{1/e}$. 4

(b) State and prove Lagrange's mean value theorem. 1+3=4

20. (a) Evaluate

(i) $\underset{x \to 1}{\mathrm{Lt}} \left(\frac{1}{x^2 - 1} - \frac{2}{x^4 - 1} \right)$

(ii) $\underset{x \to 0}{\mathrm{Lt}} (\cos x)^{\cot^2 x}$ 2+3=5

(b) Expand $\sin x$ in powers of x in infinite series stating the condition under which the expansion is valid. 3

★★★