2018/TDC/ODD/MTMG–101T/070

TDC (CBCS) Odd Semester Exam., 2018

MATHEMATICS

(1st Semester)

Course No. : MTMGEC–101T/MTMDSC–101T

(Differential Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *four* questions : 1×4=4

(a) When is the limit of a function said to exist at a point?

(b) What is the value of $\underset{x \to 0}{\text{Lt}} (1+x)^{1/x}$?

(c) Does $\underset{x \to 0}{\text{Lt}} \left\{ \sin\frac{1}{x} + x\sin\frac{1}{x} + x^2 \sin\frac{1}{x} \right\}$ exist?

(d) Find the value of

$$\underset{n\to\infty}{\text{Lt}}\left\{\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n}{n^2}\right\}$$

(e) If $\underset{x\to c}{\text{Lt}} f(x)=a$, $\underset{x\to c}{\text{Lt}} g(x)=b$, then what should be the value of $\underset{x\to c}{\text{Lt}} (fg)(x)$?

2. (a) Define Sandwich theorem. 2

Or

(b) Show by example that $\underset{x\to 0}{\text{Lt}}[f(x)+g(x)]$ and $\underset{x\to 0}{\text{Lt}} f(x)g(x)$ exist although at least one of the two limits $\underset{x\to 0}{\text{Lt}} f(x)$ and $\underset{x\to 0}{\text{Lt}} g(x)$ does not exist. 2

3. (a) (i) Show that $\underset{x\to 0}{\text{Lt}} \cos\frac{1}{x}$ does not exist. 4

(ii) Prove that

$$\underset{x\to a}{\text{Lt}} \frac{x^n-a^n}{x-a}=na^{n-1}\ \forall n$$ 4

Or

(b) (i) Show that $\underset{x\to 0}{\text{Lt}} \frac{1}{2+e^{1/x}}$ does not exist. 3

(ii) Find the value of

$$\underset{x\to 0}{\text{Lt}} \frac{\sin x^0}{x} \text{ and } \underset{x\to 0}{\text{Lt}} \frac{\sin^{-1} x}{x}$$ 3+2=5

UNIT—II

4. Answer any *four* questions : 1×4=4

(a) Does the derivative of an even function is an even function?

(b) Is the function $f(x)=\cos x$ continuous?

(c) Examine the continuity of the function

$$f(x)=\frac{1}{x-5}$$

(d) Write down the value of

$$\frac{d}{dx}(\sec^{-1} x)$$

(e) Is the function

$$f(x)=\begin{cases}\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$$

continuous at $x=0$?

5. (a) Prove that if a function f is continuous, then $|f|$ is also continuous. 2

Or

(b) If a function is continuous in an open interval, then it is bounded in that interval. Explain it with the help of an example. 2

6. (a) (i) Show that the function

$$f(x) = |x| + |x-1| + |x-2|$$

is continuous at the point $x = 0, 1, 2$. 4

(ii) Prove that if a function is differentiable at a point, then it is continuous at that point. Is the converse true? 2+2=4

Or

(b) (i) If $f(x+y) = f(x) + f(y)$ for all x and y and if $f(x)$ is continuous at $x=0$. Show that it is continuous everywhere. 4

(ii) Examine the differentiability at $x=0$ and 1 of the function defined by

$$f(x) = \begin{cases} 4-x & , x \le 0 \\ 5x+4 & ; 0 < x \le 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$$ 4

UNIT—III

7. Answer any *four* questions : 1×4=4

(a) State Euler's theorem on homogeneous function.

(b) What is the value of $D^n\left(\frac{1}{x+a}\right)$?

(c) Write down the degree of the function

$$f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$$

(d) Find y_3, if $y = e^{8x}$.

(e) Is the function $u = (x^{\frac{1}{4}} + y^{\frac{1}{4}}) / (x^{\frac{1}{5}} + y^{\frac{1}{5}})$ homogeneous?

8. (a) If $u = \cos^{-1}\left\{\frac{x+y}{\sqrt{x}+\sqrt{y}}\right\}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$ 2

Or

(b) If $y = a\cos(\log x) + b\sin(\log x)$, prove that

$$x^2 y_2 + xy_1 + y = 0$$ 2

9. (a) (i) If $\log y = \tan^{-1} x$, then show that

$$(1+x^2)y_2 + (2x-1)y_1 = 0 \text{ and}$$

$$(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0$$ 2+3=5

(ii) If $u = \log r$ and $r^2 = x^2 + y^2 + z^2$, prove that

$$r^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$$ 3

Or

(b) (i) Find y_n, if $y = \dfrac{1}{x^2 + a^2}$. 4

(ii) If V is a homogeneous function in x, y, z of degree n, prove that $\dfrac{\partial V}{\partial x}$ is a homogeneous function in x, y, z of degree $(n-1)$. 4

UNIT—IV

10. Answer any *four* questions : 1×4=4

(a) Write down the geometrical meaning of $\dfrac{dy}{dx}$.

(b) Define radius of curvature at any point on a curve.

(c) What is the condition of orthogonality of two curves $f(x, y) = 0$ and $\phi(x, y) = 0$?

(d) About which axes the curve

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

is symmetrical?

(e) Write down the equation of tangent to the curve $f(x, y) = 0$ at (x_1, y_1).

11. (a) Find the points where the tangent to the curve $y = x^3 - 3x^2 - 9x + 15$ is parallel to x-axis. 2

Or

(b) Write down the formula for finding the radius of curvature of any parametric curve. 2

12. (a) (i) Find the radius of curvature at any point (x, y) of the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. 4

(ii) What do you mean by angle between two curves? Prove that the curves $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$ and $\dfrac{x^2}{a'} + \dfrac{y^2}{b'} = 1$ will cut orthogonally if $a - b = a' - b'$. 1+3=4

Or

(b) (i) Show that at any point of the curve $x^{m+n} = k^{m-n} y^{2n}$ the mth power of the subtangent varies as the nth power of the subnormal. 4

(ii) Find the radius of curvature at any point of the curve $x = a\cos\theta$, $y = a\sin\theta$. 4