2018/TDC/ODD/MTMC–101T/068

TDC (CBCS) Odd Semester Exam., 2018

MATHEMATICS

(1st Semester)

Course No. : MTMHCC–101T

(Calculus)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) If $y = \log(x+a)$, then find y_n.

(b) If $y = e^{ax} \sin bx$, then show that

$$y_2 - 2ay_1 + (a^2 + b^2)y = 0$$

(c) State Leibnitz rule on successive differentiation.

2. Answer *either* (a) and (b) *or* (c) and (d) :

(a) If $y = \sin mx$, then show that

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} = 0$$ 3

(b) If $y = e^{a \sin^{-1} x}$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

Hence find $(y_n)_0$. 3

(c) If $x \sin\theta + y \cos\theta = a$ and
$x \cos\theta - y \sin\theta = b$

then prove that

$$\frac{d^p x}{d\theta^p} \cdot \frac{d^q y}{d\theta^q} - \frac{d^q x}{d\theta^q} \cdot \frac{d^p y}{d\theta^p}$$

is constant. 3

(d) Prove that

$$\frac{d^n}{dx^n}(x^n \sin x) = n!(P \sin x + Q \cos x)$$

where $P = 1 - \binom{n}{2}\frac{x^2}{2} + \binom{n}{4}\frac{x^4}{4!} - \cdots$

$$Q = \binom{n}{1}x - \binom{n}{3}\frac{x^3}{3!} + \binom{n}{5}\frac{x^5}{5!} - \cdots$$ 3

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Show that $\underset{x \to 0+}{\text{Lt}} (1+x)^{1/x} = e$.

(b) Define asymptote of a curve.

(c) Using the same set of rectangular axes, draw the graphs of the curves $y = \sin x$ and $y = \sin 2x$, $0 \le x \le 2\pi$.

4. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Find : 1½×2=3

(i) $\underset{x \to 0}{\text{Lt}} \frac{(e^x - 1)\tan^2 x}{x^2}$

(ii) $\underset{x \to 0}{\text{Lt}} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

(b) Examine the curve $y = x^3 - 3x + 3$ for concavity and points of inflection, if any. Hence trace the curve showing clearly the extreme points and points of reflection, if any. 3

(c) Evaluate : 1½×2=3

(i) $\underset{x \to 0}{\text{Lt}} (\cos x)^{\cot^2 x}$

(ii) $\underset{x \to 0}{\text{Lt}} \frac{xe^x - \log(1+x)}{x^2}$

(d) Find the asymptote of the curve

$$y = \frac{-8}{x^2 - 4}$$ 3

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) If $I_n = \int x^n \cos ax\, dx$

$J_n = \int x^n \sin ax\, dx$

then show that

$$aI_n = x^n \sin ax - nJ_{n-1}$$

(b) Evaluate $\int_0^{\pi/2} \cos^{10}\theta\, d\theta$.

(c) If $I_n = \int_0^{\pi/4} \tan^n \theta\, d\theta = \frac{1}{n-1} - I_{n-2}$, then find I_6.

6. Answer *either* (a) and (b) *or* (c) and (d) :

(a) From the reduction formula for $\int \cos^m x \cos nx\, dx$, obtain the value of $\int \cos^3 x \cos 5x\, dx$. 3

(b) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\, dx$, $m, n \in \mathbb{N}$, then prove that $I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$. 3

(c) Obtain a reduction formula for $\int \sec^n x\, dx$. Hence find $\int \sec^6 x\, dx$. 3

(d) Prove that

$$\int_0^{\pi/2} \cos^n x\, dx = \frac{(n-1)(n-3)\cdots 4\cdot 2}{n(n-2)\cdots 5\cdot 3}, \text{ if } n \text{ is odd}$$

$$= \frac{(n-1)(n-3)\cdots 3\cdot 1}{n(n-2)\cdots 4\cdot 2}\frac{\pi}{2}, \text{ if } n \text{ is even}$$ 3

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Find by integration the length of $y = 3x$ from $x = 0$ to $x = 3$.

(b) Find the area of the region bounded by the curve $y^2 = x$ and the line $y = x$.

(c) Show that the surface area of a sphere of radius a is $4\pi a^2$.

8. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Determine the length of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ measured from the vertex. 3

(b) Find the area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$. 3

(c) Find the length of the arc of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ lying in the 1st quadrant. 3

(d) Find the area of the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by the latus rectum about X-axis. 3

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Prove that

$$\vec{a}\times\vec{b} = \{(\hat{i}\times\vec{a})\cdot\vec{b}\}\,\hat{i} + \{(\hat{j}\times\vec{a})\cdot\vec{b}\}\,\hat{j} + \{(\hat{k}\times\vec{a})\cdot\vec{b}\}\,\hat{k}$$

(b) Find the vector equation of the plane through the point $2\hat{i}+3\hat{j}-\hat{k}$ and perpendicular to the vector $3\hat{i}+4\hat{j}+7\hat{k}$.

(c) If $\vec{r} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{s} = \sin t\hat{i} - \cos t\hat{j}$

then find the values of $\frac{d}{dt}(\vec{r}\cdot\vec{s})$ and $\frac{d}{dt}(\vec{r}\times\vec{s})$.

10. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Show that if $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar, then $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are also non-coplanar. Is this true for $\vec{a}-\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$? 3

(b) Find the vector equation of the plane passing through a given point and parallel to two given vectors. 3

(c) Show that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$ if and only if either $\vec{b}=\vec{0}$ or $\vec{c}$ is collinear with $\vec{a}$, or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$. 3

(d) Prove that the necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have a constant direction is $\vec{f}\times\frac{d\vec{f}}{dt} = \vec{0}$. 3

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2018/TDC/ODD/MTMC–102T/069

TDC (CBCS) Odd Semester Exam., 2018

MATHEMATICS

(1st Semester)

Course No. : MTMHCC–102T

(Higher Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following questions : $2\times2=4$

(a) State De Moivre's theorem for rational indices.

(b) Write down the polar form of the complex number $-1+i$.

(c) If $2\cos\theta = a + \frac{1}{a}$, $2\cos\phi = b + \frac{1}{b}$, then find the value of $ab + \frac{1}{ab}$.

2. Answer any *two* from the following questions : 5×2=10

(a) If $(a_1 + ib_1)(a_2 + ib_2)\cdots(a_n + ib_n) = A + iB$, prove that—

(i) $\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + \cdots + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$;

(ii) $(a_1^2 + b_1^2) \times (a_2^2 + b_2^2) \cdots (a_n^2 + b_n^2) = A^2 + B^2$.

(b) If $(1+x)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$ then show that—

(i) $a_0 - a_2 + a_4 - \cdots = 2^{n/2}\cos\frac{n\pi}{4}$;

(ii) $a_1 - a_3 + a_5 - \cdots = 2^{n/2}\sin\frac{n\pi}{4}$.

(c) Expand $\sin\alpha$ in ascending power of α.

(d) If $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$, then expand θ in powers of $\tan\theta$.

UNIT—II

3. Answer any *two* from the following questions : 2×2=4

(a) Define equivalence relation on a set.

(b) Let A and B be two sets. If $f : A \to B$ is one-one onto, then prove that $f^{-1} : B \to A$ is also one-one onto.

(c) Give an example of a relation which is reflexive and transitive but not symmetric.

4. Answer any *two* from the following questions : 5×2=10

(a) On the set Z of all integers, consider the relation $R = \{(a, b) : a - b \text{ is divisible by } 3\}$. Show that R is an equivalence relation. Find the partitioning of Z into mutually disjoint equivalence classes.

(b) Let $A = R - \{\frac{3}{5}\}$, $B = R - \{\frac{7}{5}\}$. Also let $f : A \to B : f(x) = \frac{7x+4}{5x-3}$ and $g : B \to A : g(y) = \frac{3y+4}{5y-7}$

Show that

$$(g \circ f) = I_A \text{ and } (f \circ g) = I_B$$ 5

(c) Let $f : R \to R : f(x) = 4x + 3$, for all $x \in R$. Show that f is invertible and find f^{-1}.

(d) If $f : A \to B$ is one-one onto, then prove that f is an invertible function.

UNIT—III

5. Answer any *two* from the following questions : 2×2=4

(a) State the principle of mathematical induction.

(b) Define Euclidean algorithm.

(c) State the fundamental theorem of arithmetic.

6. Answer any *two* from the following questions : 5×2=10

(a) Prove by principle of mathematical induction

$$1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n \cdot 2^n = (n-1)2^{n+1} + 2$$

(b) Prove by principle of mathematical induction

$$1(1!) + 2(2!) + 3(3!) + \cdots + n(n!) = (n+1)! - 1$$

(c) Using division algorithms, prove that the square of any odd integer is of the form $8k+1$ for some integer k.

(d) If $(a, 4) = 2$ and $(b, 4) = 2$, then prove that $(a+b, 4) = 4$.

UNIT—IV

7. Answer any *two* from the following questions : 2×2=4

(a) Examine the nature of the roots of the following equation by using Descartes' rule of signs $x^4 + 2x^2 + 3x - 1 = 0$.

(b) If α, β, γ be the roots of the equation

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$$

then find the value of $\sum \alpha^2\beta$.

(c) Remove the fractional coefficient of the equation

$$x^4 - \frac{2}{3}x^3 - \frac{5}{6}x^2 + \frac{7}{72}x + \frac{11}{216} = 0$$

8. Answer any *two* from the following questions : 5×2=10

(a) If α, β, γ be the roots of the equation $x^3 + px + r = 0$, then find the value of $\sum \frac{1}{\alpha^2 - \beta\gamma}$.

(b) Solve the following equation by Cardan's method :

$$x^3 - 6x - 4 = 0$$

(c) Solve the equation

$$27x^3 + 42x^2 - 28x - 8 = 0$$

whose roots are in geometric progression.

(d) If α, β, γ be the roots of $2x^3 + x^2 + x + 1 = 0$, then find the equation whose roots are

$$\frac{1}{\beta^3} + \frac{1}{\gamma^3} - \frac{1}{\alpha^3}, \frac{1}{\gamma^3} + \frac{1}{\alpha^3} - \frac{1}{\beta^3}, \frac{1}{\alpha^3} + \frac{1}{\beta^3} - \frac{1}{\gamma^3}$$

UNIT—V

9. Answer any *two* from the following questions : 2×2=4

(a) Define rank of a matrix.

(b) Under what condition the rank of the matrix

$$\begin{bmatrix} 2 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$

is 3?

(c) If A is a non-zero column matrix and B is a non-zero row matrix, then show that rank $(AB) = 1$.

10. Answer any *two* from the following questions : 5×2=10

(a) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{pmatrix}$$

by reducing it to echelon form.

(b) Solve the following system of equations by Gaussian elimination method :

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

(c) Prove that the interchange of a pair of rows does not alter the rank of a matrix.

(d) Reduce the following matrix to normal form :

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{pmatrix}$$

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