2024/TDC (CBCS)/EVEN/SEM/
MTMGEC-601T/241

TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(6th Semester)

Course No. : MTMGEC-601T

(Differential Equation)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *four* questions : 1×4=4

(a) Write the condition of exactness of the differential equation $Mdx + Ndy = 0$, where M and N are functions of x and y.

(b) What is meant by an integrating factor?

(c) Determine the integrating factor of

$$(x^4 + y^4)dx - xy^3dy = 0$$

(d) Is the differential equation $x\,dy + y\,dx = 0$ exact?

(e) Solve $\sin x \cos y\,dy + \cos x \sin y\,dx = 0$.

2. Answer any *one* question : 2

(a) Solve $(y - px)(p - 1) = p$.

(b) Show that the equation

$$(y^2 + 2xy)\,dx - x^2\,dy = 0$$

is not exact.

3. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Solve : 4

$$xy(p^2 + 1) = (x^2 + y^2)\,p$$

(b) Solve : 4

$$x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$$

(c) Solve : 4

$$y = 2px + p^2 y$$

(d) Solve : 4

$$(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$$

UNIT—II

4. Answer any *four* of the following as directed : 1×4=4

(a) Write a linear ordinary differential equation of second-order with constant coefficients.

(b) Define auxiliary equation.

(c) Verify that $\cos x$ is a solution of

$$\frac{d^2y}{dx^2} + y = 0$$

(d) If $y(x)$ be a solution of

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$$

satisfying $y(x_0) = 0$ and $y'(x_0) = 0$ for some $x_0 \in (a, b)$, then $y(x) =$ _____.

(Fill in the blank)

(e) An integral belonging to complementary function of $y'' + Py' + Qy = 0$, where P, Q are functions of x or constants is _____ if $1 + P + Q = 0$.

(Fill in the blank)

5. Answer any *one* question : 2

(a) If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are two solutions of differential equation $y'' + 9y = 0$, then show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions.

(b) Prove that $y = e^x$ is a part of CF of the equation $xy'' - (2x-1)y' + (x-1)y = 0$.

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Show that linearly independent solutions of $y'' - 2y' + 2y = 0$ are $e^x \sin x$ and $e^x \cos x$. What is the general solution? 4

(b) If $f_1(x)$ and $f_2(x)$ are two solutions of

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$$

then prove that $c_1 f_1(x) + c_2 f_2(x)$ is also a solution of this equation, where c_1 and c_2 are arbitrary constants. 4

(c) Given that x, x^2 and x^4 are all solutions of

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0$$

Show that they are linearly independent on the interval $0 < x < \infty$ and write the general solution. 4

(d) Solve : 4

$$xy'' - (x+2)y' + 2y = 0$$

UNIT—III

7. Answer any *four* of the following as directed : 1×4=4

(a) Write a third-order homogeneous linear differential equation.

(b) Find the complementary function of

$$(D^2 + 1)y = e^x \sin x$$

(c) Define particular integral for the linear differential equation with constant coefficients $f(D)y = X$, X is a function of x or constant.

(d) Find the particular integral of

$$\frac{d^2y}{dx^2} - y = e^{2x}$$

(e) If v be any solution of a given non-homogeneous nth order linear differential equation and u be any solution of the corresponding homogeneous equation, then $u + v$ is also a solution of the given non-homogeneous equation.

(Write True or False)

8. Answer any *one* question : 2

(a) Solve :

$$\frac{d^3y}{dx^3} - 13\frac{dy}{dx} - 12y = 0$$

(b) Find the complete solution of

$$(D^2+1)^2 y = \cos 3x$$

9. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Solve the initial value problem

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0, \quad y(0) = 3, \quad y'(0) = 5$$ 4

(b) Using the method of variation of parameter, solve

$$\frac{d^2y}{dx^2} + 9y = \sec 3x$$ 4

(c) Find the complete solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \sin x$$ 4

(d) Given that $y = x$ is a solution of

$$(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

Find a linearly independent solution by reducing the order. 4

UNIT—IV

10. Answer any *four* of the following as directed : 1×4=4

(a) What is meant by a Cauchy-Euler equation?

(b) Write the condition for exactness of the differential equation

$$P\,dx + Q\,dy + R\,dz = 0$$

where P, Q, R are the functions of x, y, z.

(c) Give an example of a homogeneous total differential equation.

(d) The system of differential equations

$$\frac{dx}{dt} = 2x - y - 5t$$

$$\frac{dy}{dt} = 3x + 6y - 4$$

is an example of homogeneous system.

(Write True or False)

(e) For $F_1dx + F_2dy + F_3dz = 0$, write the auxiliary equations. (F_1, F_2 and F_3 are the functions of x, y, z)

11. Answer any *one* question : 2

(a) Solve :

$$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{x}$$

(b) Reduce

$$x^2\frac{d^2y}{dx^2} + y = 3x^2$$

to linear differential equation with constant coefficients.

12. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Solve : 4

$$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=2x^2$$

(b) Solve : 4

$$\frac{dx}{dt}+2x-3y=t$$
$$\frac{dy}{dt}-3x+2y=e^{2t}$$

(c) Verify that the equation

$$(2x^2+2xy+2xz^2+1)\,dx+dy+2z\,dz=0$$

is integrable and solve it. 4

(d) Solve : 4

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$$

UNIT—V

13. Answer any *four* questions : 1×4=4

(a) Write a second-order linear partial differential equation.

(b) Define degree of a partial differential equation.

(c) Is the partial differential equation

$$\frac{\partial z}{\partial y}=3\left(\frac{\partial z}{\partial x}\right)^2$$

linear?

(d) What is the order of the partial differential equation

$$y\left\{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right\}=z\frac{\partial z}{\partial y}?$$

(e) Define non-linear partial differential equation.

14. Answer any *one* question : 2

(a) Find the order and degree of

$$\left(1+\frac{\partial^2 z}{\partial x^2}\right)^{3/2}=\left(1+\frac{\partial z}{\partial y}\right)^{1/2}$$

(b) Eliminate a and b from $z=a(x+y)+b$ to form the partial differential equation.

15. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Form a partial differential equation by eliminating functions f and g from

$$z=f(x+iy)+g(x-iy)$$ 4

(b) Find the differential equation of all spheres of radius 4 with centres on the *xy*-plane. 4

(c) For each of the following partial differential equations, determine whether the equation is hyperbolic, parabolic or elliptic : 2+2=4

(i) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

(ii) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$

(d) Find a partial differential equation by eliminating *a*, *b*, *c* from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

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