TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(6th Semester)

Course No. : MTMDSE-602T

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* from Option—A *or* Option—B

OPTION—A

Course No. : MTMDSE-602T (A)

(**Hydrodynamics**)

UNIT—I

1. Answer any *four* as directed : 1×4=4

   (a) Define real fluid. Give one example.

   (b) What do you mean by laminar flow?

(c) Define rotational flow.

(d) In which type of flow velocity potential exists?

(i) Rotational flow

(ii) Irrotational flow

(Choose the correct answer)

(e) The equation of streamline is

(i) $\vec{q} \times d\vec{r} = \vec{0}$

(ii) $\vec{q} \cdot d\vec{r} = 0$

(Choose the correct answer)

2. Answer any *one* of the following questions : 2

(a) Describe briefly Eulerian method of describing fluid motion.

(b) If velocity components are given by

$$u = -\frac{k^2 x}{x^2 + y^2}, \quad v = \frac{k^2 y}{x^2 + y^2}, \quad w = 0$$

show that fluid motion is irrotational.

3. Answer any *one* of the following questions : 8

(a) Obtain the equations of the streamlines and the path lines when velocity components are

$$u = \frac{x}{1+t}, \quad v = \frac{y}{1+t}, \quad w = \frac{z}{1+t}$$

(b) If the velocity of an incompressible fluid motion at the point (x, y, z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5} \right)$$

prove that the velocity potential is $\frac{\cos\theta}{r^2}$. Also determine the streamlines, given that $r^2 = x^2 + y^2 + z^2$ and $z = r\cos\theta$.

UNIT—II

4. Answer any *four* as directed : 1×4=4

(a) Equation of continuity signifies the principle of _____.

(Fill in the blank)

(b) If $u = kx$, $v = ky$, $w = -2kz$ are the velocity components of an incompressible flow, is the motion possible?

(c) Write the equation of continuity in spherical polar coordinates.

(d) What is the equation of continuity in cylindrical coordinates?

(e) Write the vector equation of the equation of continuity.

5. Answer any *one* of the following questions : 2

(a) Obtain the equation of continuity of an incompressible fluid from Euler's equation of continuity (vector form).

(b) Derive the equation of continuity in Lagrangian form.

6. Answer any *one* of the following questions : 8

(a) Derive the equation of continuity in Cartesian coordinates.

Or

Derive the equation of continuity (vector form) by Euler's method.

(b) If the lines of motion are curves on the surfaces of cones having their vertices at the origin and the axis of z for common surface, prove that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho u) + \frac{2\rho u}{r} + \frac{\operatorname{cosec} \theta}{r} \frac{\partial}{\partial \phi}(\rho w) = 0$$

UNIT—III

7. Answer any *four* as directed : 1×4=4

(a) If $\vec{q}$ is the fluid velocity at time t, what is the acceleration?

(b) What does local derivative mean?

(c) Write the components of acceleration in Cartesian coordinates.

(d) Define stream function.

(e) Current function is also known as _____.

(Fill in the blank)

8. Answer any *one* of the following questions : 2

(a) Determine the components of acceleration in x-direction, given the velocity field

$$\vec{q} = (Axy^2t)\hat{i} + (Bx^2yt)\hat{j} + (Cxyz)\hat{k}$$

(b) Show that the stream function satisfies Laplace's equation.

9. Answer any *one* of the following questions :

(a) *(i)* Show that acceleration of a fluid particle is the material derivative of its fluid velocity. 5

(ii) Given, $\frac{D\vec{f}}{Dt} = \frac{\partial \vec{f}}{\partial t} + (\vec{q} \cdot \vec{\nabla})\vec{f}$. What do $\frac{D}{Dt}$, $\frac{\partial}{\partial t}$, $(\vec{q} \cdot \vec{\nabla})$ signify? 3

(b) *(i)* Given the velocity field

$$\vec{q} = (Ax^2y)\hat{i} + (By^2zt)\hat{j} + (Czt^2)\hat{k}$$

Determine the acceleration of a fluid particle of fixed identity. 5

(ii) The velocity components of a flow are given by

$$u = 2Axy, \quad v = A(a^2 + x^2 - y^2)$$

Determine the stream function. 3

UNIT—IV

10. Answer any *four* as directed : 1×4=4

(a) Equation of motion signifies the principle of ____.

(Fill in the blank)

(b) Write Euler's equation of motion in z-direction.

(c) Define conservative force.

(d) What does the equation $\frac{d}{dt}(T+W) = R$ represent?

(i) Equation of continuity

(ii) Equation of motion

(iii) Energy equation

(Choose the correct answer)

(e) Write Lamb's hydrodynamical equation.

11. Answer any *one* of the following questions : 2

(a) Derive Euler's equation of motion in Cartesian form from vector form.

(b) Write the statement of energy equation for an incompressible fluid.

12. Answer any *one* of the following questions : 8

(a) Derive Euler's equation of motion in the form

$$\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p$$

(b) Derive Lamb's hydrodynamical equation.

UNIT—V

13. Answer any *four* as directed : 1×4=4

(a) Bernoulli's equation is obtained by ____ Euler's equation of motion.

(Fill in the blank)

(b) If the motion is steady, velocity potential does not exist and V the potential function from which the external forces are derivable, then Bernoulli's theorem is ____.

(Fill in the blank)

(c) The Bernoulli's equation for unsteady and irrotational motion is given by

(i) $-\frac{\partial \phi}{\partial t}+\frac{q^2}{2}+V+\frac{p}{\rho}=F(t)$

(ii) $-\frac{\partial \phi}{\partial t}+\frac{q^2}{2}+V=F(t)$

(iii) $-\frac{\partial \phi}{\partial t}-\frac{q^2}{2}+V-\frac{p}{\rho}=0$

(iv) $\frac{q^2}{2}+V+\frac{p}{\rho}=F(t)$

(Choose the correct answer)

(d) A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. If a side tube is connected with the pipe at the formal place, water will be sucked up through it into the pipe from a reservoir at a depth h below the pipe, s being the delivery per second, where h is given by

(i) $\frac{s^2}{2g}\left(\frac{1}{A^2}+\frac{1}{B^2}\right)$

(ii) $\frac{s^2}{2g}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$

(iii) $\frac{2g}{s^2}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$

(iv) $\frac{2g}{s^2}\left(\frac{1}{A^2}+\frac{1}{B^2}\right)$

(Choose the correct answer)

(e) If the fluid be homogeneous and incompressible, then in usual symbols, the Bernoulli's theorem becomes

(i) $\frac{q^2}{2}+V+p=C$

(ii) $q^2+V+\frac{p}{\rho}=C$

(iii) $\frac{q^2}{2}+V+\frac{p}{\rho^2}=C$

(iv) $\frac{q^2}{2}+V+\frac{p}{\rho}=C$

(Choose the correct answer)

14. Answer any *one* of the following questions : 2

(a) State D' Alembert's paradox.

(b) Explain Euler's momentum theorem.

15. Answer any *one* of the following questions : 8

(a) State and prove Bernoulli's theorem.

(b) A stream is rushing from a boiler through a conical pipe, the diameter of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion be supposed to be that of the divergence from the vertex of the cone, prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/2K}$$

where K is the pressure divided by the density and supposed constant.

OPTION—B

Course No. : MTMDSE–602T (B)

(Theory of Equation)

UNIT—I

1. Answer any *four* of the following questions : 1×4=4

(a) Write down the degree of the polynomial $x^2(x+1)+(1-x^2)^2$.

(b) If A and B are two polynomials of degree 4 and 5 respectively, then what should be the degree of the polynomial AB?

(c) Write Descartes' rule of sign for negative roots.

(d) When is a polynomial said to be a monic polynomial?

(e) Write down an example of a non-polynomial expression.

2. Answer any *one* of the following questions : 2

(a) Find the minimum value of the polynomial $4x^2 - 6x + 1$.

(b) Find the maximum number of negative real zeroes of $2x^4 + x^3 - 6x^2 - 7x + 1$.

3. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Show that the equation $x^3 + 2x^2 - 2x - 1 = 0$ has one positive root and two negative roots one lying between − 3 and − 1 and another lying between − 1 and 0. 4

(b) Find the maximum and minimum values of the polynomial $-x^2 + x + 2$. 4

(c) Show that the minimum value of $y = x^x$ is $e^{-1/e}$. 4

(d) Apply Descartes' rule of sign to find the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$. 4

UNIT—II

4. Answer any *four* of the following questions : 1×4=4

(a) Give one example of symmetric function.

(b) Form the equation whose roots are m times of the roots of the equation

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \ldots + P_n = 0$$

(c) If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2$.

(d) Find the equation whose roots are reciprocal of the roots of $x^2 - 5x + 6 = 0$.

(e) Transform the equation whose roots are same but change in sign of the roots of $x^2 - px + q = 0$.

5. Answer any *one* of the following questions : 2

(a) If α, β, γ be the roots of the equation $x^2 + qx + r = 0$, then find the value of $\Sigma(\alpha^2 - \beta\gamma)$.

(b) If α, β, γ be the roots of $x^3 - px^2 + qx + r = 0$, then form the equation whose roots are $\alpha + m$, $\beta + m$, $\gamma + m$.

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Solve the equation

$$4x^4 - 4x^3 - 13x^2 + 9x + 9 = 0$$

given that the sum of two roots is zero. 4

(b) Find the equation whose roots are the squares of the roots of the equation

$$x^4 - 2x^3 + 3x^2 - x + 7 = 0$$ 4

(c) Find an equation whose roots are the roots of the equation

$$x^7 + 3x^5 + x^3 - x^2 + 7x + 2 = 0$$

with their signs changed. 3

(d) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of

$$\left(\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}\right)\left(\frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}\right)$$ 5

UNIT—III

7. Answer any *four* of the following questions : $1\times4=4$

(a) Write down one example of a biquadratic equation.

(b) What do you mean by derived function?

(c) State Newton's theorem.

(d) Give one example of a standard form of cubic equation.

(e) Write down the standard form of a biquadratic equation.

8. Answer any *one* of the following questions : 2

(a) If a, b, c be the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of

$$\Sigma\left(\frac{b}{c}+\frac{c}{b}\right)$$

(b) How can you remove the second term from the equation

$$ax^3 + 3bx^2 + 3cx + d = 0, \quad a \neq 0\ ?$$

9. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Solve $x^3 - 6x - 9 = 0$ by Cardon's method. 3

(b) Solve the equation $x^4 - 2x^2 + 8x - 3 = 0$. 5

(c) Solve $x^3 + 6x^2 - 12x + 32 = 0$. 4

(d) Solve the equation

$$6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0$$ 4

UNIT—IV

10. Answer any *four* of the following questions : $1\times4=4$

(a) Define superior limit of roots.

(b) State Sturm's theorem.

(c) Find a superior limit of the positive roots of the equation

$$2x^5+6x^4-2x^3+8x^2-51x+18=0$$

(d) What do you mean by Sturm's function?

(e) Find the first derived function $f_1(x)$, where $f(x)=3x^3-9x$.

11. Answer any *one* of the following questions : 2

(a) Find the Sturm's functions for the equation $x^2-3x+5=0$.

(b) Find the nature of the roots of

$$x^4-5x^3+9x^2-7x+2=0$$

12. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Find the number and position of the real roots of $x^3-7x+7=0$. 4

(b) Apply Budan's method to separate the roots of the equation

$$x^4-8x^3+69x^2-70x-42=0$$ 4

(c) Apply Sturm's theorem to analyze the equation

$$x^4-4x^3+7x^2-6x-4=0$$ 4

(d) Calculate Sturm's functions and locate the position of the real roots of the equation $x^3-3x-1=0$. 4

UNIT—V

13. Answer any *four* of the following questions : 1×4=4

(a) Find the condition of real and equal roots of $bx^2+2cx+a=0$.

(b) Give one example of a numerical equation.

(c) Why are numerical methods used?

(d) What is the difference between an algebraic equation and a numerical equation?

(e) How many zeroes does a biquadratic equation have?

14. Answer any *one* of the following questions : 2

(a) Find the roots α, β, γ of the equation $x^3-6x^2+5=0$, when α, β are connected by $2\alpha=3\beta$.

(b) What do you mean by numerical equation?

15. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) Solve the equation

$$x^4 - 4x^3 - 4x^2 - 4x - 5 = 0$$

given that two roots α, β are connected by the relation $\alpha\alpha + \beta = 3$. 4

(b) Find all the roots of the equation $x^4 - 2x^3 - 19x^2 + 68x - 60 = 0$ lying between − 6 and 6. 4

(c) Prove that the roots of the equation

$$x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$$

are all real and solve it when two of the quantities *a*, *b*, *c* become equal. 4

(d) Show that the two real roots of the equation $x^4 - 4x^3 - 3x + 23 = 0$ lies in (2, 3) and (3, 4). 4

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