**TDC (CBCS) Even Semester Exam., 2024**

# MATHEMATICS

**( 6th Semester )**

Course No. : MTMDSE-601T

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

Candidates have to answer *either* from Option—A *or* Option—B *or* Option—C

## OPTION—A

Course No. : MTMDSE-601T (A)

**( Complex Analysis )**

*Full Marks* : 70
*Pass Marks* : 28

### UNIT—I

**1.** Answer any *four* of the following questions : 1×4=4

(a) Find the argument of the complex number

$$\frac{1+2i}{1-(1-i)^2}$$

(b) Find the modulus of

$$\frac{1+i\sqrt{3}}{\sqrt{3}+1}$$

(c) If $\alpha$ is the $n$th root of unity other than 1, then find the value of $1+\alpha+\alpha^2+\cdots+\alpha^{n-1}$.

(d) Write the locus of the point $z$, where $|z|<1$.

(e) What is the area of the triangle formed by the complex numbers $z$, $iz$ and $z+iz$?

2. Answer any *one* of the following questions : 2

(a) Find the equation of the circle through the points 1, $i$, $1+i$.

(b) If $|z-2+i|\le 2$, then find the greatest and the least value of $|z|$.

3. Answer *either (a) or (b)* : 8

(a) (i) Show that $\arg z+\arg\bar{z}=2n\pi$. 4

(ii) Prove that the area of the triangle whose vertices are the points $z_1$, $z_2$, $z_3$ on the Argand diagram is

$$\sum\left[\frac{(z_2-z_3)|z_1|^2}{4iz_1}\right]$$ 4





(b) (i) Give the geometrical interpretation of

$$\arg\left(\frac{z-\alpha}{z-\beta}\right)$$ 4

(ii) Prove that a triangle with vertices $z_1$, $z_2$ and $z_3$ is equilateral if

$$z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$$ 4

UNIT—II

4. Answer any *four* of the following questions : 1×4=4

(a) Define the continuity of a function $f(z)$ at a point $z=z_0$.

(b) What is an analytic function?

(c) Write Cauchy-Riemann equations in polar form.

(d) Is the function $u=y^3-3x^2y$ harmonic?

(e) Give an example of a function $f(z)$, which is continuous at a point but is not differentiable at that point.

5. Answer any *one* of the following questions : 2

(a) Show that an analytic function in a domain with its derivative zero for every point of the domain is constant.

(b) Show that an analytic function with constant modulus in a domain is constant.

6. Answer *either* (a) *or* (b) : 8

(a) (i) Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at that point. 4

(ii) If $f(z)=u+iv$ is an analytic function of $z=x+iy$ and $u-v=e^x(\cos y-\sin y)$, find $f(z)$ in terms of $z$. 4

(b) (i) If $f(z)$ is an analytic function of $z$, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\mathrm{Re}\, f(z)|^2=2|f'(z)|^2$$ 4

(ii) Show that the function $f(z)=e^{-z^{-4}}$ $(z\neq 0)$ and $f(0)=0$ is not analytic at $z=0$ although Cauchy-Riemann equations are satisfied at that point. 4





UNIT—III

7. Answer any *four* of the following questions : 1×4=4

(a) Define a rectifiable curve.

(b) If $C$ is given by the equation $|z-a|=r$, then what is the value of

$$\int_C \frac{dz}{z-a}?$$

(c) Define a simply connected region.

(d) If $f(z)$ is an analytic function in a simply connected domain $D$ and if $C$ is any closed continuous rectifiable curve in $D$, then what is the value of

$$\int_C f(z)\,dz\,?$$

(e) If $f(z)$ is an analytic function of $z$ and $f'(z)$ is continuous at each point within and on a closed contour $C$, then what is the value of

$$\int_C f(z)\,dz\,?$$

8. Answer any *one* of the following questions : 2

(a) Find the value of the integral

$$\int_0^{1+i}(x-y+ix^2)\,dz$$

along the straight line from $z=0$ to $z=1+i$.

(b) Evaluate the integral

$$\int_0^{1+i} z^2\,dz$$

**9.** Answer *either* (a) *or* (b) : 8

(a) (i) If $f(z)$ is an analytic function of $z$ and $f'(z)$ is continuous at each point within and on a closed contour $C$, then prove that

$$\int_C f(z)\,dz = 0$$ 4

(ii) If $C$ is a rectifiable curve of length $l$ and $|f(z)| \le M$, $\forall z \in C$, then prove that

$$\left|\int_C f(z)\,dz\right| \le Ml$$ 4

(b) (i) Evaluate

$$\int_C |z|\,dz$$

where $C$ is the circle $|z-1|=1$, described in the positive sense. 4

(ii) If $f(z)$ is analytic in a simply connected domain $D$, then prove that the integral along every rectifiable curve in $D$ joining any two given points of $D$ is the same. 4

UNIT—IV

**10.** Answer any *four* of the following questions : 1×4=4

(a) State Cauchy's integral formula.

(b) If a function $f(z)$ is analytic in a domain $D$ and $z = z_0$ is a point of $D$, then what is $f'(z_0)$?





(c) Evaluate $\int_C \frac{dz}{z-1}$, where $C$ is the circle $|z|=2$.

(d) Evaluate $\int_C \frac{z\,dz}{(z-1)^2}$, where $C$ is the circle $|z|=3$.

(e) If $C$ is the circle $|z-a|=r$, then for what value of $n$

$$\int_C \frac{dz}{(z-a)^n}$$

is equal to $2\pi i$?

**11.** Answer any *one* from the following questions : 2

(a) Evaluate $\int_C \frac{dz}{z(z+\pi i)}$, where $C$ is $|z+3i|=1$.

(b) Evaluate $\int_C \frac{e^{iz}}{z-\pi i}dz$, where $C$ is the ellipse $|z-2|+|z+2|=6$.

**12.** Answer *either* (a) *or* (b) : 8

(a) (i) State and prove Morera's theorem. 1+4=5

(ii) Evaluate

$$\int_C \frac{z\,dz}{(9-z^2)(z+i)}$$

where $C$, is the circle $|z|=2$ described in positive sense. 3

(b) (i) If $f(z)$ is analytic within a circle $C$, given by $|z-a|=R$ and if $|f(a)|\leq M$ on $C$, then prove that

$$|f^n(a)| \leq \frac{Mn!}{R^n}$$ 5

(ii) Evaluate

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where $C$ is the circle $|z|=3$. 3

UNIT—V

**13.** Answer any *four* of the following questions : 1×4=4

(a) State Taylor's theorem for complex functions.

(b) What is Laurent's series?

(c) Define an entire function.

(d) Write the series for $e^z$.

(e) Is the function $f(z)=\cos z$, $z \in C$ bounded?





**14.** Answer any *one* from the following questions : 2

(a) Expand $\frac{1}{z}$ as a Taylor's series about $z=1$.

(b) Find the zeros of $z^3-3z^2+z-3$.

**15.** Answer *either (a) or (b)* : 8

(a) (i) State and prove Liouville's theorem. 1+4=5

(ii) Find the Taylor's series which represents the function

$$\frac{z^2-1}{(z+2)(z+3)}$$

when $|z|<2$. 3

(b) (i) State and prove the fundamental theorem of algebra. 1+4=5

(ii) Obtain expansion for

$$\frac{(z+2)(z-2)}{(z+1)(z+4)}$$

which is valid for the region $|z|<1$. 3

OPTION—B

Course No. : MTMDSE–601T (B)

**( Linear programming )**

*Full Marks* : 70
*Pass Marks* : 28

UNIT—I

1. Answer any *four* of the following questions : 1×4=4

*(a)* Define line segment joining two points in Euclidean space $\mathbb{E}^n$.

*(b)* Define convex combination of *r* point in $\mathbb{E}^n$.

*(c)* What are surplus variables?

*(d)* Write the LPP in standard form :

$$\text{Max } Z = x_1 + 2x_2 + x_3$$

subject to

$$x_1 + 3x_2 - x_3 \le 5$$
$$2x_1 - 3x_2 + x_3 \ge 7$$
$$x_1, x_2, x_3 \ge 0$$

*(e)* What is the maximum number of basic solutions of a system of 3 linear equations in 6 unknowns?





2. Answer any *one* of the following questions : 2

*(a)* Show that a hyperplane is a convex set.

*(b)* Give example with justification to show that the union of two convex sets may not be convex.

3. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 8

*(a)* Solve graphically : 5

$$\text{Min } Z = x + 2y$$

subject to

$$x + y \ge 6$$
$$3x + y \ge 9$$
$$x - y \le 3$$
$$x, y \ge 0$$

*(b)* Prove that

$$S = \{(x, y) \in \mathbb{E}^2 \mid x^2 + y^2 \le 10\}$$

is a convex set in $\mathbb{E}^2$. 3

*(c)* A toy company manufactures two types of dolls *A* and *B*, the profits on each doll being ₹ 4 and ₹ 6 respectively. Each doll of type *B* requires twice as long to manufacture as that of each type *A* doll. The company has sufficient raw materials to produce 1000 dolls per day. The type *B* doll requires a fancy dress

and only 400 such dresses are available per day. If the company manufactures only type *A* dolls, then it could produce 1500 dolls per day. Formulate this problem as an LPP of suitable type addressing all the constraints. 4

*(d)* Find all the basic solutions of the system of equations :

$$3x_1 + 2x_2 - 3x_3 + x_4 = 2$$
$$6x_1 + 4x_2 - x_1 + 2x_4 = 3$$ 4

UNIT—II

**4.** Answer any *four* of the following questions : 1×4=4

*(a)* In a simplex table, what is the condition under which a maximization LPP has unbounded solution?

*(b)* What type of LPP can be solved by simplex method?

*(c)* What is the auxiliary objective function in a two-phase method?

*(d)* In artificial variable technique what is the condition under which an LPP has no feasible solution?

*(e)* What is the modified objective function in Big-*M* method?





**5.** Answer any *one* of the following questions : 2

*(a)* Construct the initial simplex table for the LPP :

Max $Z = 5x_1 + 3x_2 - x_3$

subject to

$$x_1 - x_2 + x_3 \le 5$$
$$2x_1 - x_2 + 3x_3 \ge -3$$
$$x_1, x_2, x_3 \ge 0$$

*(b)* Write a short note on Big-*M* method.

**6.** Answer *either (a) or (b)* : 8

*(a)* Solve by simplex method :

Min $Z = x_1 - 3x_2 + 2x_3$

subject to

$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

*(b)* Solve by Big-*M* method :

Max $Z = 4x_1 + 3x_2$

subject to

$$x_1 + x_2 \le 50$$
$$x_1 + 2x_2 \ge 80$$
$$3x_1 + 2x_2 \ge 140$$
$$x_1, x_2 \ge 0$$

UNIT—III

7. Answer any *four* of the following questions : 1×4=4

(a) What is duality?

(b) If the *i*th constraint in the primal is an equality, what can you conclude about the *i*th variable in the dual?

(c) What do you mean by an unbalanced transportation problem?

(d) What is penalty in Vogel's method?

(e) How is an unbalanced transportation problem made balanced in order to find an initial basic feasible solution?

8. Answer any *one* of the following questions : 2

(a) Write a short note on North-West corner rule.

(b) Describe in brief how can we find an initial basic feasible solution to an unbalanced transportation problem.

9. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 8

(a) Write the dual of the LPP : 4

$$\text{Max } Z = x_1 + 2x_2 + x_3$$

subject to

$$x_1 + x_2 - x_3 \le 5$$
$$4x_1 - x_2 - 2x_3 \le 7$$
$$2x_1 + x_2 - 3x_3 = 8$$
$$x_1,\ x_2,\ x_3 \ge 0$$





(b) Find an initial basic feasible solution using matrix minima method : 4

*Destination*

| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | *Availability* |
|---|---|---|---|---|---|---|
| *Sources* | $S_1$ | 2 | 3 | 1 | 4 | 15 |
| | $S_2$ | 1 | 2 | 4 | 3 | 10 |
| | $S_3$ | 3 | 4 | 1 | 2 | 25 |
| | | 10 | 20 | 12 | 8 | |

(c) Describe the correspondence between the primal and its dual. 2

(d) Find an initial basic feasible solution to the following unbalanced transportation problem using Vogel's method : 6

*Destination*

| | | $D_1$ | $D_2$ | $D_3$ | *Availability* |
|---|---|---|---|---|---|
| *Sources* | $S_1$ | 5 | 1 | 3 | 10 |
| | $S_2$ | 3 | 2 | 4 | 20 |
| | $S_3$ | 1 | 4 | 5 | 30 |
| | $S_4$ | 6 | 2 | 7 | 40 |
| *Requirement* | | 20 | 30 | 30 | |

## UNIT—IV

**10.** Answer any *four* of the following questions : 1×4=4

*(a)* What should be the number of positive allocations in an $m \times n$ transportation problem in order to perform optimality test?

*(b)* When is an initial basic feasible solution in a transportation problem said to be degenerate?

*(c)* In MODI method, what is the cell evaluation for an occupied cell?

*(d)* In an assignment problem how many allocations can be made in each row?

*(e)* Name a method to solve an assignment problem.

**11.** Answer any *one* of the following questions : 2

*(a)* Give a brief description of MODI method for obtaining optimal solution of a transportation problem with non-degenerate basic feasible solution.

*(b)* Write the mathematical formulation of an assignment problem in form of an LPP.





**12.** Answer *either (a) or (b)* : 8

*(a)* Find an initial basic feasible solution of the following transportation problem and use MODI method to obtain an optimal solution :

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ | |
|---|---|---|---|---|---|
| $O_1$ | 1 | 2 | 1 | 4 | 30 |
| $O_2$ | 3 | 3 | 2 | 1 | 50 |
| $O_3$ | 4 | 2 | 5 | 9 | 20 |
| | 20 | 40 | 30 | 10 | |

*(b)* Solve the assignment problem :

| *Men*→ *Job*↓ | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| *I* | 15 | 13 | 14 | 17 |
| *II* | 11 | 12 | 15 | 13 |
| *III* | 13 | 12 | 10 | 11 |
| *IV* | 15 | 17 | 14 | 16 |

## UNIT—V

**13.** Answer any *four* of the following questions : 1×4=4

*(a)* What is a two-person zero-sum game?

*(b)* What is saddle point?

*(c)* What is mixed strategy?

*(d)* What is a payoff matrix?

*(e)* What is meant by the value of a game?

**14.** Answer any *one* of the following questions : 2

*(a)* Two players *A* and *B* match coins. If the coins match, *A* wins two units of value. If the coins do not match, then *B* wins two units of value. Construct the payoff matrix.

*(b)* Find the saddle point of the payoff matrix :

$$A \begin{matrix} & \begin{matrix} & B & \\ I & II & III \end{matrix} \\ \begin{matrix} I \\ II \end{matrix} & \begin{bmatrix} 6 & 8 & 6 \\ 2 & 12 & 4 \end{bmatrix} \end{matrix}$$

**15.** Answer *either (a) or (b)* : 8

*(a)* Solve the game graphically whose payoff matrix is

$$A \begin{matrix} & \begin{matrix} & B & \\ I & II & III \end{matrix} \\ \begin{matrix} I \\ II \end{matrix} & \begin{bmatrix} -4 & 2 & -6 \\ 3 & -9 & 4 \end{bmatrix} \end{matrix}$$





*(b)* Solve by using any appropriate method the game whose payoff matrix is

$$A \begin{matrix} & \begin{matrix} & & B & \\ I & II & III & IV \end{matrix} \\ \begin{matrix} I \\ II \\ III \\ IV \end{matrix} & \begin{bmatrix} 3 & 2 & 4 & 0 \\ 2 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{matrix}$$

OPTION—C

Course No. : MTMDSE–601T (C)

**( Object-Oriented Programming in C++ )**

*Full Marks* : 50
*Pass Marks* : 20

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

   *(a)* What is C++? How is C++ different from C?

   *(b)* Compare the OOP language and structured programming language.

   *(c)* If $a = 100$ and $b = 4$, then determine the result of the following :

      *(i)* $a+ = b$

      *(ii)* $a\% = b$

2. Answer any *one* of the following questions : 6

   *(a)* Explain the principles of object-oriented programming.

   *(b)* Explain the structure of C++ program with example.





UNIT—II

3. Answer any *two* of the following questions : 2×2=4

   *(a)* How to declare a pointer to an object?

   *(b)* Write any two advantages of inheritance.

   *(c)* Define reference variable. Give its syntax.

4. Answer any *one* of the following questions : 6

   *(a)* Demonstrate encapsulation and polymorphism.

   *(b)* How can a pointer be declared and initialized? Give an overview of pointer arithmetic.

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

   *(a)* What is the difference between a constructor and a destructor?

   *(b)* Explain the difference between copy constructor and assignment operator.

   *(c)* What are abstract classes and how do they different from a class?

**6.** Answer any *one* of the following questions : 6

*(a)* Write a C++ program to calculate the roots of a quadratic equation by initializing the object using default constructor.

*(b)* How to define a class in C++? How to declare objets for the class? Give an example.

## UNIT—IV

**7.** Answer any *two* of the following questions : 2×2=4

*(a)* State the use of scope resolution operator in C++.

*(b)* What is class function? What is class declaration?

*(c)* What is friend function and what is the use of friend function?

**8.** Answer any *one* of the following questions : 6

*(a)* Write a C++ program to multiply the private members of two classes using a friend function.

*(b)* A friend function cannot be used to overload the assignment operator =. Explain why. When is a friend function compulsory?





## UNIT—V

**9.** Answer any *two* of the following questions : 2×2=4

*(a)* What is the difference between function overloading and function template?

*(b)* What is the need of overloading operators and functions?

*(c)* Write a short note on namespaces.

**10.** Answer any *one* of the following questions : 6

*(a)* Write C++ program to overload '+' operator to add two matrices.

*(b)* Write down the rules for overloading operators.

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