

**2024/TDC (CBCS)/EVEN/SEM/
MTMSEC-601T/242**

TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(6th Semester)

Course No. : MTMSEC-601T

(Analytical Geometry)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any three of the following as directed : 1×3=3
- (a) If the origin is transferred to the point (h, k) without changing direction of the axes, then write the transformation formula for the translation of axes.
- (b) What does the equation $x^2 - y^2 = 0$ become when the origin is transferred to the point $(-1, 2)$?

(2)

- (c) A homogeneous second degree equation always represents a pair of straight lines passing through origin.

(Write True or False)

- (d) Write down the equation of bisectors of angles between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$.

2. Answer any one question :

2

- (a) Transform to axes inclined at 30° to the original axes the equation

$$x^2 + 2\sqrt{3}xy - y^2 - 2 = 0$$

- (b) Show that $3x^2 + 5xy + 2y^2 = 0$ represents a pair of straight lines. Find the straight lines.

3. Answer any one question :

5

- (a) Find the angle by which the axes should be rotated so that the equation $ax^2 + 2hxy + by^2 = 0$ becomes another equation in which the xy term is absent.

Also find the angle through which the axes are to be rotated so that the equation $17x^2 + 18xy - 7y^2 = 1$ may be reduced to the form $Ax^2 + By^2 = 1$, $A > 0$. Find also A and B .

2+3=5

(3)

- (b) Find the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$. Also write the condition of coincidence and perpendicularity of the lines. 3+2=5

UNIT—II

4. Answer any three of the following questions :

1×3=3

- (a) Obtain the equation of the circle whose center is at $(-1, -2)$ and radius is $\sqrt{2}$.
 (b) Define orthogonal circles.
 (c) Write down the equation of circles which intersect two circles $x^2 + y^2 + 2x + 1 = 0$ and $x^2 + y^2 + 2y + 3 = 0$.
 (d) Find the radical axis of the two circles

$$x^2 + y^2 + 4x - 2y + 9 = 0$$

and

$$x^2 + y^2 + 2x + 3y - 5 = 0.$$

5. Answer any one question :

2

- (a) Find the value of λ for which the circles

$$x^2 + y^2 + \lambda x + 3y - 5 = 0$$

and

$$x^2 + y^2 + 5x + xy + 7 = 0$$

cut each other orthogonally.

(4)

- (b) Show that the straight line $lx + my + n = 0$ touches the circle $x^2 + y^2 = a^2$ if $n^2 = a^2(l^2 + m^2)$.

6. Answer any one question : 5

- (a) (i) Find the equation of the circle which passes through origin and cut orthogonally the circles

$$x^2 + y^2 - 8y + 12 = 0$$

and

$$x^2 + y^2 - 4x - 6y - 3 = 0 \quad 3$$

- (ii) Find the radical center of the set of circles

$$x^2 + y^2 + x + 2y + 3 = 0, x^2 + y^2 + 2x + 4y + 5 = 0$$

$$\text{and } x^2 + y^2 - 7x - 8y - 9 = 0. \quad 2$$

- (b) If two tangents drawn from a point to a conic be perpendicular to one another, then prove that the locus of their point of intersection is a circle. 5

UNIT—III

7. Answer any three of the following questions :

1×3=3

- (a) Define focal chord of a parabola.
 (b) Find the foci of the ellipse $9x^2 + 25y^2 = 225$.

(5)

- (c) Write down the condition that the line $y = mx + c$ is a tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- (d) Write the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in parametric form.

8. Answer any one question : 2

- (a) Find the equation of the ellipse which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$.

- (b) Find the points on the conic $\frac{14}{r} = 3 - 8\cos\theta$, whose radius vector is 2.

9. Answer any one question : 5

- (a) (i) Obtain polar equation of a conic referred to a focus as pole. 3
 (ii) Write polar equation of the ellipse

$$\frac{x^2}{36} + \frac{y^2}{20} = 1,$$

if the pole be at its right-hand focus and the positive direction of the x -axis be the positive direction of the polar axis. 2

(6)

- (b) Prove that the straight line $lx + my + n = 0$ is a normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

if

$$\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{n^2}$$

5

UNIT—IV

10. Answer any three of the following as directed : 1×3=3

- (a) Under what condition

$$ax^2 + by^2 + cz^2 + 2gx + 2fy + 2hz + c = 0$$

represents a sphere?

- (b) The section of a sphere by a plane represents a ____.

(Fill in the blank)

- (c) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 49$ at the point $(6, -3, -2)$.

- (d) What is the equation of a sphere which passes through origin having intercepts a , b and c on the axes?

(7)

11. Answer any one question :

2

- (a) Find the radius of the circle $x^2 + y^2 + z^2 = 25$, $x + 2y + 2z + 9 = 0$.

- (b) Find the values of c for which the plane $x + y + z = c$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$$

12. Answer any one question :

5

- (a) Show that the length of the shortest distance between the straight lines

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5} \text{ and } \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

is $4\sqrt{3}$ units and the equations of the line of shortest distance are $x = y = z$.

5

- (b) (i) A plane passing through a fixed point (a, b, c) cuts the axes in A , B and C . Show that the locus of the center of the sphere $OABC$ is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

3

- (ii) Find the equation of the circle on the sphere $x^2 + y^2 + z^2 = 49$ whose center is at the point $(2, -1, 3)$.

2

(8)

UNIT—V

13. Answer any *three* of the following questions :

1×3=3

- (a) What do you mean by guiding curve of a cone?
- (b) Define axis and semi-vertical angle of a cone.
- (c) Define axis of a cylinder.
- (d) What do you mean by generator of a cylinder?

14. Answer any *one* question :

2

- (a) Find the equation of the cone whose vertex is at the origin and whose axis is $\frac{x}{3} = \frac{y}{2} = \frac{z}{4}$ and semi-vertical angle 45° .
- (b) Find the equation of the cylinder generated by the lines parallel to $\frac{x}{1} = \frac{y}{-2} = \frac{z}{5}$, the guiding curve being the conic $x=0, y^2=8z$.

15. Answer any *one* question :

5

- (a) Prove that $ax+by+cz=0$ ($abc \neq 0$), cuts the cone $yz+zx+xy=0$ is perpendicular straight lines if

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

5

(9)

- (b) (i) Find the equation of a cylinder whose guiding curve is represented by $f(x,y)=0, z=0$ and whose generators are parallel to the line

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

3

- (ii) Find the equation of the cylinder generated by the straight lines parallel to z -axis and passing through the curve of intersection of the plane $lx+my+nz=p$ and the surface $ax^2+by^2+cz^2=1$.

2
