2024/TDC (CBCS)/EVEN/SEM/ MTMSEC–401T (A/B/C)/236

TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(4th Semester)

Course No. : MTMSEC–401T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* from Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MTMSEC–401T (A)

(Graph Theory)

UNIT—I

1. Answer any *three* of the following : $1\times3=3$

(a) Define a graph.

(b) Define a simple graph.

(c) What is a complete graph?

(d) What is a bipartite graph?

2. Answer any *one* of the following : 2

(a) Define isolated and pendant vertices.

(b) Sketch the complete graph K_5.

3. Answer *either (a) or (b)* : 5

(a) Define a complete bipartite graph. Show that a complete bipartite graph $K_{a,b}$ has ab edges.

(b) Show that the sum of the degrees of the vertices of a graph is twice the number of its edges.

UNIT—II

4. Answer any *three* of the following : 1×3=3

(a) Define cut point.

(b) What is a tree?

(c) What is a leaf in a tree?

(d) What is a forest?

5. Answer any *one* of the following : 2

(a) Show that in a tree, there is exactly one path between every pair of its vertices.

(b) What is a bridge in a connected graph? Illustrate with a suitable example.

6. Answer *either (a) or (b)* : 5

(a) "A connected graph G is bipartite if and only if G has no odd cycles." Prove it.

(b) Show that a tree with n vertices has $n-1$ edges.

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) Define Hamiltonian graph.

(b) Define Eulerian graph.

(c) Define isomorphism between two graphs.

(d) What is the order of the adjacency matrix of a graph with 4 vertices?

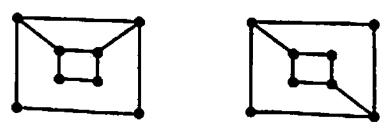
8. Answer any *one* of the following : 2

(a) Define traversable graphs with suitable example.

(b) Define traceable graphs with suitable example.

9. Answer *either (a) or (b)* : 5

(a) Are the following graphs isomorphic? Justify your answer :



(b) Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) Define planar graphs.

(b) Give an example of a non-planar graph.

(c) For which n, the complete graph K_n is planar?

(d) State a necessary and a sufficient condition for a graph to be planar.

11. Answer any *one* of the following : 2

(a) Prove or disprove :

The complete bipartite graph $K_{3,3}$ is planar.

(b) If G is a planar graph with no parallel edges on n vertices and e edges, where $e \geq 3$, then show that $e \leq 3n - 6$.

12. Answer *either (a) or (b)* : 5

(a) State and prove Euler theorem on plane graphs.

(b) Show that a graph is planar if and only if it can be embedded on a sphere.

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) What is the travelling salesman's problem?

(b) What is Floyd-Warshall algorithm used for?

(c) Write one use of Dijkstra's algorithm.

(d) Give an example of a weighted graph.

14. Answer any *one* of the following : 2

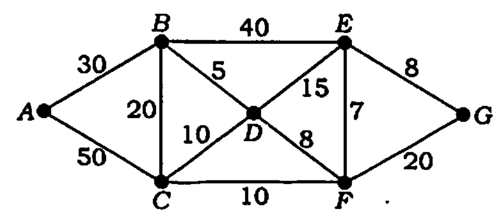
(a) Write the steps of Floyd-Warshall algorithm.

(b) Write the steps of Dijkstra's algorithm.

15. Answer *either (a) or (b)* : 5

(a) Illustrate Floyd-Warshall algorithm with a suitable example.

(b) Find a shortest path from A to G for the following graph :



OPTION—B

Course No. : MTMSEC–401T (B)

(Special Functions)

UNIT—I

1. Answer any *three* of the following : 1×3=3

   (a) Write down the Legendre's polynomial of first kind of order n.

   (b) Write down the differential equation of the Legendre's equation.

   (c) What are the roots of $P_n(x)=0$, where $P_n(x)$ is a Legendre's polynomial of first kind?

   (d) Fill in the blank :

   $$P_n(-1) = \_\_\_\_ .$$

2. Answer any *one* of the following : 2

   (a) Express $2-3x+4x^2$ in terms of Legendre's polynomial.

   (b) Show that $P_n'(1)=\frac{1}{2}n(n+1)$.

3. Answer *either (a) or (b)* : 5

   (a) Prove that

   $$1+\frac{1}{2}P_1(\cos\theta)+\frac{1}{3}P_2(\cos\theta)+\ldots=\log\left[\frac{(1+\sin\frac{\theta}{2})}{\sin\frac{\theta}{2}}\right]$$

   (b) When n is positive integer, then prove that

   $$P_n(x)=\frac{1}{\pi}\int_0^\pi [x\pm\sqrt{(x^2-1)}\cos\phi]^n\,d\phi$$

   Hence find $P_n(\cos\theta)$.

UNIT—II

4. Answer any *three* of the following : 1×3=3

   (a) Write the Bessel's function of first kind of order n.

   (b) What is the complete solution of the Bessel's differential equation?

   (c) What is the relation between $J_n(x)$ and $J_{-n}(x)$, n being a positive integer?

   (d) Write down the Bessel's differential equation of order 0.

5. Answer any *one* of the following : 2

   (a) Prove that $J_{-1/2}(x)=\sqrt{\left(\frac{2}{\pi x}\right)}\cdot\cos x$.

   (b) Prove that

   $$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$$

6. Answer *either* (a) or (b) : 5

(a) Prove the following recurrence formulae for $J_n(x)$: 3+2=5

(i) $xJ_n' = -nJ_n + xJ_{n-1}$

(ii) $J_{n-1} - J_{n+1} = 2J_n'$

(b) Show that if n is a positive integer

$$J_{-n}(x) = (-1)^n J_n(x)$$

Also show that

$$J_{+1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$$ 3+2=5

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) Find the Laplace transform of t^n, n is a positive integer.

(b) Write the first translation or shifting property of Laplace transform.

(c) If $L\{F(t)\} = f(s)$, then what is $L\{t^n F(t)\}$, where $n = 1, 2, 3, \ldots$?

(d) What is inverse Laplace transform of $\dfrac{s}{s^2 - a^2}$?

8. Answer any *one* of the following : 2

(a) Evaluate $L\{(t^2+1)^2\}$; where $L\{F(t)\} = f(s)$.

(b) Evaluate $L^{-1}\left\{\dfrac{1}{s^2 - 2s + 2}\right\}$, where

$$L^{-1}\{f(s)\} = F(t)$$

9. Answer *either* (a) or (b) : 5

(a) If $F(t) = t^2$, $0 < t < 4$ and $F(t+4) = F(t)$, then find $L\{F(t)\}$.

(b) Evaluate $L^{-1}\left\{\dfrac{1}{s(s^2+9)}\right\}$, where $L^{-1}\{f(s)\} = F(t)$.

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) If $L\{F(t)\} = f(s)$, then what is the value of $L\{F''(t)\}$?

(b) Write True or False :

An ODE is converted to an algebraic equation when we use Laplace transform.

(c) If $L\{y(t)\} = y(s)$, then obtain Laplace transform of $y''(t) + y'(t) + y(t)$.

(d) Write one application of Laplace transform in real-life problem.

11. Answer any *one* of the following : 2

(a) If $u = u(x, t)$ is a function of two variables x and t, then what is *(i)* $L\left\{\frac{\partial u}{\partial t}\right\}$, *(ii)* $L\left\{\frac{\partial^2 u}{\partial t^2}\right\}$; $L[u(x, t)] = \bar{u}(x, s)$?

(b) Solve the following ODE by Laplace method :

$$\frac{dy}{dt} + y = 1; \;\; y(0) = 1; \;\; L\{y(t)\} = \bar{y}$$

12. Answer *either (a) or (b)* : 5

(a) Solve the differential equation

$$(D^2 + 9)y = \cos(2t)$$

if $y(0) = 1$, $y'\left(\frac{\pi}{2}\right) = -1$.

(b) Use Laplace transform method to solve

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t$$

if $y = 2$, $\frac{dy}{dt} = -1$ at $t = 0$.

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) Write the formula for infinite Fourier sine transformation of $f(x)$.

(b) Write Fourier cosine integral formula.

(c) Write the relation between Laplace and Fourier transforms.

(d) Write Fourier exponential integral formula.

14. Answer any *one* of the following : 2

(a) If

$$f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x \geq a \end{cases}$$

then find Fourier cosine transformation of $f(x)$.

(b) If $f(x) = e^{-5x} + 3e^{-3x}$, then find Fourier sine transformation of $f(x)$.

15. Answer *either (a) or (b)* : 5

(a) Using Fourier transform, show that

$$\int_0^\infty \frac{\cos sx}{s^2 + 1} ds = \frac{\pi}{2} e^{-x} \quad (x \geq 0)$$

(b) Using Fourier integral, show that

$$e^{-ax} = \frac{2a}{\pi} \int_0^\infty \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda \quad (a > 0, \; x \geq 0)$$

OPTION—C

Course No. : MTMSEC-401T (C)

(Vector Calculus / Vector Analysis)

UNIT—I

1. Answer any *three* of the following : 1×3=3

(a) What is the condition for coplanarity of three vectors?

(b) Find

$$(\hat{i}+\hat{j}-6\hat{k})\cdot\{(\hat{i}-3\hat{j}+4\hat{k})\times(2\hat{i}-5\hat{j}+3\hat{k})\}$$

(c) Write the vector equation of the line passing through $\vec{a}$ and parallel to $\vec{b}$.

(d) What is the value of $\vec{a}\times(\vec{b}\times\vec{c})$?

2. Answer any *one* of the following : 2

(a) Show that

$$(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})=[\vec{a}, \vec{b}, \vec{c}]\vec{c}$$

(b) Show that

$$[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}]=2[\vec{a}, \vec{b}, \vec{c}]$$

3. Answer *either (a) or (b)* : 5

(a) Find the vector equation of a plane in parametric form, passing through a given vector and parallel to two vectors. Also find the equation of the plane in non-parametric form. 3+2=5

(b) Prove the following : 2+3=5

(i) $\vec{a}\times(\vec{b}\times\vec{a})=(\vec{a}\times\vec{b})\times\vec{a}$

(ii) $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})+(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})+(\vec{c}-\vec{d})\cdot(\vec{a}-\vec{b})=0$

UNIT—II

4. Answer any *three* of the following : 1×3=3

(a) Define continuity of a vector function at a point.

(b) Write down the formula for $\frac{d}{dt}(\vec{u}\cdot\vec{v})$.

(c) Prove that the derivative of a constant vector is a zero vector.

(d) If $\vec{r}=\sin t\hat{i}+\cos t\hat{j}+2t\hat{k}$, then find $\left|\frac{d\vec{r}}{dt}\right|$.

5. Answer any *one* of the following : 2

(a) If $\vec{r} = \vec{a}\cos nt + \vec{b}\sin nt$, then prove that

$$\frac{d^2\vec{r}}{dt^2} + n^2\vec{r} = 0$$

(b) If $\vec{A}$ and $\vec{B}$ are differential vector functions, then prove that

$$\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$$

6. Answer *either (a) or (b)* : 5

(a) Prove that a vector function $\vec{f}(t)$ has constant magnitude iff $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$.

(b) If $\vec{r} = (3t,\ 3t^2,\ 2t^3)$, then show that

$$[\dot{\vec{r}}\ \ddot{\vec{r}}\ \dddot{\vec{r}}] = 216$$

UNIT—III

7. Answer any *three* of the following : 1×3=3

(a) Define a scalar point function.

(b) Define a vector point function.

(c) What is the value of grad f ?

(d) Define an irrotational vector.

8. Answer any *one* of the following : 2

(a) If $u = x^2 - y^2 + 4z$, then find $\nabla^2 u$.

(b) Show that $\vec{\nabla} \cdot \vec{r} = 3$.

9. Answer *either (a) or (b)* : 5

(a) Prove that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \nabla^2 \vec{a}$$

(b) Find divergence and curl of

$$\text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$$

UNIT—IV

10. Answer any *three* of the following : 1×3=3

(a) Write the value of

$$\int \left(2\vec{r} \cdot \frac{d\vec{r}}{dt} \right) dt$$

(b) Define line integral of a vector function.

(c) If $\vec{a} = 2u^4\hat{i} - 5\hat{j} + (u - u^2)\hat{k}$, then find $\int \vec{a}\, du$.

(d) Write the value of

$$\int \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right) dt$$

11. Answer any *one* of the following : 2

(a) Evaluate $\int_0^2 \vec{f}(t)\,dt$, if

$$\vec{f}(t) = t\hat{i} + (t^2 - 2t)\hat{j} + (3t^2 + 3t^3)\hat{k}$$

(b) Integrate the function $\vec{F} = x^2\hat{i} - xy\hat{j}$ from (0, 0) to (1, 1) along $y^2 = x$.

12. Answer *either (a) or (b)* : 5

(a) If

$$\vec{A} = 2t^2\hat{i} + 3(t-1)\hat{j} + 4t^2\hat{k}$$
$$\vec{B} = (t-1)\hat{i} + t^2\hat{j} + (t-2)\hat{k}$$

then find—

(i) $\int_0^2 (\vec{A}\cdot\vec{B})\,dt$;

(ii) $\int_0^2 (\vec{A}\times\vec{B})\,dt$. 2+3=5

(b) Prove the necessary and sufficient condition that the vector field defined by the vector point function $\vec{F}$ with continuous derivatives be conservative is that curl $\vec{F} = \vec{\nabla}\times\vec{F} = 0$.

UNIT—V

13. Answer any *three* of the following : 1×3=3

(a) What is the formula for tangential component of acceleration?

(b) State the principle of conservation of energy.

(c) Define kinetic energy.

(d) State the principle of work.

14. Answer any *one* of the following : 2

(a) A particle moves along the curve $x = 4\cos t$, $y = 4\sin t$, $z = 6t$. Find the velocity of the particle at time $t = 0$.

(b) Find the work done by the force $\vec{F} = (0, 0, -mg)$ in moving a particle of mass m from $O(0, 0, 0)$ to $A(1, 1, 1)$ along the curve $x = t$, $y = t^2$, $z = t^3$, t being a parameter.

15. Answer *either (a) or (b)* : 5

(a) A particle moves along the curve

$$x = t^3 + 1,\ y = t^2,\ z = 2t + 5$$

where t is the time. Find the magnitudes of the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$.

(b) A moving particle of mass 3 units has position vector $\vec{r}$ at time t as given by

$$\vec{r} = (t^2 - 3t)\hat{i} + \left(\frac{1}{2}t^2 - 1\right)\hat{j} - \frac{1}{2}t^2\hat{k}$$

Prove that the resultant force $\vec{F}$ acting on the particle is given by

$$\vec{F} = 6\hat{i} + 3\hat{j} - 3\hat{k}$$

Also find the kinetic energy about the origin for the particle at any time t.

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