2024/FYUG/EVEN/SEM/
MATDSC-151T/126

FYUG Even Semester Exam., 2024

MATHEMATICS

(2nd Semester)

Course No. : MATDSC-151T

(Analytical Geometry)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. What does the equation $x^2 - y^2 + 2x + 4y = 0$ become when the origin is transferred to the point $(-1, 2)$?

2. Find the equation to which the equation $x^2 - y^2 - 2ax + 2by + c^2 = 0$ will reduce when origin is changed to the point (a, b) and the axes are turned through an angle $\frac{\pi}{2}$ in the positive direction.

3. Show that the equation

$$x^2+6xy+9y^2+4x+12y+4=0$$

represents a pair of coincident straight lines.

4. Find the radical axis for the pair of circles

$$x^2+y^2-3x-4y+5=0$$
$$3x^2+3y^2-7x+8y+11=0$$

5. Find the condition that the line $y=mx+c$ may touch the parabola $y^2=4ax$.

6. Prove that the pair of circles

$$x^2+y^2+ax-by+c=0$$
$$x^2+y^2+bx+ay-c=0$$

cuts orthogonally.

7. Find the polar of the point (1, 1) with respect to the conic $x^2+xy+3y^2+4x+6y-10=0$.

8. Find the pole of the straight line $9x+y-28=0$ with respect to the circle $2x^2+2y^2-3x+5y-7=0$.

9. Find the points on the conic $\frac{15}{r}=1-4\cos\theta$ whose radius vector is 5.

10. Find the radius of the circle

$$x^2+y^2+z^2-2y-4z-11=0,\ x+2y+2z=15$$

11. Find the equation of the sphere with centre at the point (1, –1, 3) and touches the plane

$$2x+y-3z=5$$

12. Find the equation of the sphere which passes through the origin and cuts the axes at distances a, b and c respectively.

13. Define *(a)* right circular cone and *(b)* generator of a cylinder.

14. Find the equation of the cone with vertex at origin and a given guiding curve $f(x, y)=0$, $z=c$.

15. Find equation of right circular cylinder whose axis is $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$ and radius is 2.

SECTION—B

Answer any *five* of the following questions : 10×5=50

16. *(a)* The equation

$$3x^2+2xy+3y^2-18x-22y+50=0$$

is transformed to $4x^2+2y^2=1$ when referred to rectangular axes through the point (2, 3). Find the inclination of the latter axes to the former. 5

(b) Show that the equation $ax^2+2bxy+by^2=0$ represents a pair of straight lines passing through the origin. Find the angle between them. 5

17. *(a)* Find the equation of bisectors of the angles between the lines represented by

$$ax^2+2hxy+by^2=0$$ 4

(b) If, by a rotation of rectangular axes about the origin, the expression $(ax+by)$ becomes $(a'x'+b'y')$, then prove that $a^2+b^2=a'^2+b'^2$. 4

(c) Through what angle must the axes be turned to remove xy term from $7x^2+4xy+3y^2$? 2

18. *(a)* Find the equation of the circle which cuts orthogonally the three circles

$$x^2+y^2=16;\ x^2+y^2-14x+40=0$$
$$x^2+y^2-12y+32=0$$ 4

(b) Prove that the straight line $lx+my+n=0$ touches the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

if $n^2=a^2l^2-b^2m^2$. 4

(c) Find the radical centre of the three circles

$$x^2+y^2+x+2y+3=0$$
$$x^2+y^2+2x+4y+5=0$$
$$x^2+y^2-7x+8y+9=0$$ 2

19. *(a)* Find the radical axis of the circles

$$x^2+y^2+2ax+c^2=0$$
$$x^2+y^2+2by+c^2=0$$

and hence show that the condition that they touch each other is

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4c^2}$$ 4

(b) Find the condition that the line $y=mx+c$ is a tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ 4

(c) Find the equation of the circle through the points of intersection of the circles

$$x^2+y^2+2x+3y-7=0$$
$$x^2+y^2+3x-2y-1=0$$

and through the point (1, 2).

20. *(a)* If the polar of a point (α, β) with respect to the parabola $y^2=4ax$ touches the circle $x^2+y^2=a^2$, then prove that the point (α, β) lies on the hyperbola

$$4x^2-y^2=4a^2$$ 4

(b) Obtain the polar equation of a conic referred to a focus as pole. 4

(c) Change $r=2a\cos\theta$ into Cartesian form. 2

21. *(a)* If the pole of the straight line with respect to the circle $x^2+y^2=a^2$ lies on $x^2+y^2=k^2a^2$, then prove that the straight line will touch the circle

$$x^2+y^2=\frac{a^2}{k^2}$$ 4

(b) If PSP' and QSQ' be two perpendicular focal chords of a conic, prove that $\frac{1}{SP\cdot SP'}+\frac{1}{SQ\cdot SQ'}$ is constant. 4

(c) Write down the equation of a chord joining two points whose vectorial angles are $(\alpha+\beta)$ and $(\alpha-\beta)$. Also write the equation of the tangent at any point whose vectorial angle is α. 2

22. *(a)* Find the shortest distance between the straight lines $\frac{x-3}{-3}=\frac{y-8}{1}=\frac{z-3}{-1}$ and $\frac{x+3}{3}=\frac{y+7}{-2}=\frac{z-6}{-4}$ and the equations of the line of shortest distance. 5

(b) A sphere of constant radius r passes through the origin and cuts the axes in A, B, C. Prove that the locus of the foot of the perpendicular from o to the plane ABC is given by

$$(x^2+y^2+z^2)^2(x^{-2}+y^{-2}+z^{-2})=4r^2$$ 5

23. *(a)* Find the equations of the tangent planes to the sphere

$$x^2+y^2+z^2-2x-4y-6z+2=0$$

parallel to the plane $x-y-z=0$. 3

(b) Find the equation of the sphere for which the circle

$$x^2+y^2+z^2+7y-2z+2=0$$
$$2x+3y+4z=8$$

is a great circle.

(c) Obtain the shortest distance between the straight line

$$ax+by+cz+d=0=a'x+b'y+c'z+d'$$

and the axis of z. 3

24. *(a)* Find the equation of right circular cone with vertex at (1, –2, –1), semi-vertical angle 60° and the axis is

$$\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z+1}{5}$$ 4

(b) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=9,\ z=1$. 4

(c) Find the equation of the cone whose vertex is the origin and which passes through the curve of intersection

$$x^2+y^2=2z,\ ax+by+cz=1$$ 2

25. *(a)* The plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

meets the coordinate axes at A, B, C. Find the equation of the cone generated by the straight lines drawn from O to meet the circle ABC. 4

(b) Find the equation of right circular cylinder whose axis is the line passes through the point (1, 3, 4) and has 1, –2, 3 as its direction ratios and radius equal to 3. 4

(c) Find the equation of the cylinder whose generators are parallel to the y-axis and which passes through the curve of intersection $x^2+y^2+z^2=3$ and $x-y+z=3$. 2

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