TDC (CBCS) Even Semester Exam., 2024

MATHEMATICS

(2nd Semester)

Course No. : MTMDSC/GEC–201T

(Differential Equation)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *four* of the following questions : $1\times4=4$

(a) Is the differential equation $(x^2+y^2+x)dx+xydy=0$ exact?

(b) If $Mdx+Ndy=0$ is homogeneous and $Mx+Ny\neq0$, then what is the integrating factor of $Mdx+Ndy=0$?

(c) What is Clairaut's equation?

(d) If

$$\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(x)$$

then what is the integrating factor of $Mdx+Ndy=0$?

(e) Solve $(p-1)(p-2)=0$, where $p=\frac{dy}{dx}$.

2. Answer any *one* of the following questions : 2

(a) Solve :

$$(2x-y+1)dx+(2y-x+1)dy=0$$

(b) Solve :

$$(p^3-6p^2+11p-6)=0,\ p=\frac{dy}{dx}$$

3. Answer any *one* of the following questions : 8

(a) (i) Solve : 4

$$xdx+ydy=a^2\frac{xdy-ydx}{x^2+y^2}$$

(ii) Solve $y+px=p^2x^4$, where $p=\frac{dy}{dx}$. 4

(b) (i) Solve : 4

$$\frac{xdy-ydx}{x^2+y^2}+xy^2dx+x^2ydy=0$$

(ii) Reduce the differential equation $(px-y)(x-yp)=2p$ to Clairaut's form by the substitution $x^2=u$, $y^2=v$ and find its complete solution, where $p=\frac{dy}{dx}$. 4

UNIT—II

4. Answer any *four* of the following questions : 1×4=4

(a) Give an example of second-order linear differential equation with constant coefficient.

(b) Define linearly dependent solution.

(c) Verify that $y=e^{4x}$ is one of the solutions of

$$\frac{d^3y}{dx^3}-64y=0$$

(d) Find the value of $W(e^x, e^{2x})$.

(e) Verify that $y = \sin x$ is the solution of

$$\frac{d^2y}{dx^2} + y = 0$$

5. Answer any *one* of the following questions : 2

(a) Show that $\sin x$ and $\sin x - \cos x$ are linearly independent solutions of

$$\frac{d^2y}{dx^2} + y = 0$$

(b) Show that the Wronskian of $e^{2x}\cos 4x$ and $e^{2x}\sin 4x$ is $4e^{4x}$.

6. Answer any *one* of the following questions : 8

(a) (i) Consider the linear differential equation $y''(x) + Py'(x) + Qy(x) = 0$, where P and Q are either constants or functions of x alone. Then prove that two solutions of the differential equation are linearly dependent iff their Wronskian is zero. 5

(ii) Consider the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

Show that e^{2x} and e^{3x} are linearly independent solutions of this equation on the interval $-\infty < x < \infty$. Also write the general solution of the given equation. 3

(b) (i) Consider the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0$$

Show that the functions e^x, e^{4x} and $2e^x - 3e^{4x}$ are solutions of this equation. Are the solutions e^{4x} and $2e^x - 3e^{4x}$ linearly independent? 4

(ii) Prove that e^{-x}, e^{3x} and e^{4x} are all solutions of

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

Show that they are linearly independent on the interval $-\infty < x < \infty$ and write the general solution. 4

UNIT—III

7. Answer any *four* of the following questions : 1×4=4

(a) Solve :

$$\frac{d^2y}{dx^2} - 8y = 0$$

(b) Solve :

$$\frac{d^2y}{dx^2} = e^{4x}$$

(c) Find the CF of the differential equation $(D^2 - 5D + 6)y = e^{3x}$, where $D = \frac{d}{dx}$.

(d) Find the PI of the differential equation $(D^2 - 1)y = \cos 2x$, where $D = \frac{d}{dx}$.

(e) Let the given differential equation be $\phi(D)y = f(x)$. If $f(x) = e^{\alpha x}V$, where V is a function of x, then write its particular integral.

8. Answer any *one* of the following questions : 2

(a) Solve $(D^2 + 4)y = e^x$, $D = \frac{d}{dx}$.

(b) Solve $(D^2 + 1)y = 0$, given that $y = 2$, when $x = 0$ and $y = -2$, when $x = \pi/2$; $D = \frac{d}{dx}$.

9. Answer any *one* of the following questions : 8

(a) (i) Solve the equation

$$\frac{d^2y}{dx^2} = a + bx + cx^2$$

given that $\frac{dy}{dx} = 0$, when $x = 0$ and $y = d$, when $x = 0$. 4

(ii) Solve $(D^3 + 1)y = \cos 2x$, $D = \frac{d}{dx}$. 4

(b) (i) Solve : 4

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \sin 2x$$

(ii) Using method of variation of parameters, solve

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$$ 4

UNIT—IV

10. Answer any *four* of the following questions : 1×4=4

(a) Define total differential equation.

(b) What do you mean by Cauchy-Euler equation?

(c) Give an example of Cauchy-Euler differential equation of second order.

(d) What is the geometrical interpretation of

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}?$$

(e) Is the equation $y\,dx + x\,dy + z\,dy = 0$ integrable?

11. Answer any *one* of the following questions : 2

(a) Show that $(yz + z^2)dx - xz\,dy + xy\,dz = 0$ is integrable.

(b) Solve :

$$\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}$$

12. Answer any *one* of the following questions : 8

(a) (i) Solve : 4

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x \log x$$

(ii) Solve : 4

$$(yz + xyz)\,dx + (zx + xyz)\,dy + (xy + xyz)\,dz = 0$$

(b) (i) Solve : 4

$$\frac{dx}{dt} = -\omega y, \quad \frac{dy}{dt} = \omega x$$

(ii) Solve : 4

$$\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$$

UNIT—V

13. Answer any *four* of the following questions : 1×4=4

(a) What do you mean by partial differential equation?

(b) Define order of a partial differential equation.

(c) Whether the partial differential equation

$$\frac{\partial z}{\partial y} = x\left(\frac{\partial z}{\partial x}\right) + \left(\frac{\partial z}{\partial x}\right)^2$$

is linear or non-linear?

(d) Write the order of the partial differential equation

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4z$$

(e) Under what condition, a partial differential equation will be linear?

14. Answer any *one* of the following questions : 2

(a) Form a partial differential equation by eliminating a and b from $z = ax + a^2y^2 + b$.

(b) Form a partial differential equation by eliminating arbitrary function from $z = f(x^2 + y^2)$.

15. Answer any *one* of the following questions : 8

(a) (i) Form the partial differential equation of all spheres whose centres lie on the x-axis. 4

(ii) Obtain the partial differential equation by eliminating the arbitrary function f from the relation $z = f\left(\frac{xy}{z}\right)$. 4

(b) (i) Obtain the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$. 4

(ii) Form a partial differential equation of all planes cutting equal intercepts from the X and Y axes. 4

★★★

2024/TDC (CBCS)/EVEN/SEM/
MTMDSC/GEC–201T/231