2023/TDC(CBCS)/ODD/SEM/
PHSHCC–301T/151

TDC (CBCS) Odd Semester Exam., 2023

PHYSICS

(Honours)

(3rd Semester)

Course No. : PHSHCC–301T

(Mathematical Physics—II)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. Distinguish between even function and odd function citing example for each.

2. Write the complex form of the Fourier series.

3. Write down the Parseval identity of Fourier transform explaining the terms involved therein.

UNIT—II

4. Explain what you understand by regular and irregular singular points.

5. State the conditions for which $x = x_0$ will be a regular singular point and an irregular singular point for the differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

6. What do you understand by indicial equation obtained during power series solution of ODE around regular singularity?

UNIT—III

7. Write the Rodrigues formula for Legendre polynomial. What is the orthogonality condition of the Legendre polynomial?

8. Write down the generating function for Bessel function. Hence find $J_0(x)$.

9. Find the value of $P_n(1)$.

UNIT—IV

10. Show that

$$\frac{\beta(m+1),n}{m} = \frac{\beta(m, n+1)}{n} = \frac{\beta(m, n)}{m+n}$$

11. Show that $\beta(m, n)$ is symmetric in m and n.

12. Show that Dirac delta function is a symmetric function.

UNIT—V

13. Write down Laplace's equation in spherical polar coordinates.

14. Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x \partial y} = \cos(2x + 3y)$$

15. Find the solution of the diffusion equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 0 \cdot 25y = 0$$

Given $y = 0$ at $x = 0$ and

$$\frac{dy}{dx} = 1 \text{ at } x = 0$$

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. A periodic function of period 2π is defined as

$$f(x) = x^2, \ -\pi \le x \le \pi$$

Expand $f(x)$ in Fourier series and show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

3+3=6

17. Write down orthogonality conditions for sine and cosine functions. Find the conditions for Fourier coefficients. 2+4=6

UNIT—II

18. Write down Legendre's differential equation and check the nature at $x = 0$. Hence obtain power series solution of it. 2+4=6

19. Write down Laguerre differential equation and solve it by Frobenius method. 6

UNIT—III

20. Prove the following recurrence relations for Legendre polynomial : 3+3=6

(a) $(2n+1)x P_n(x) = (n+1) P_{n+1}(x) + x P_{n-1}(x)$

(b) $nP_n(x) = xP'_n(x) - P'_{n-1}(x)$

21. If a and b are different roots of $J_n(x) = 0$, then show that

$$\int_0^1 xJ_n(ax) J_n(bx) = 0 \quad \text{for} \quad a \ne b$$

$$= \frac{1}{2}[J'_n(a)]^2 \quad \text{for} \quad a = b$$

6

UNIT—IV

22. (a) Starting with the fundamental definition of gamma function, show that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(b) Prove that

$$\Gamma\left(\frac{1}{2} - n\right) \Gamma\left(\frac{1}{2} + n\right) = (-1)^n \pi$$

4+2=6

23. (a) Explain how Dirac delta function can be expressed as a limit of Gaussian function.

(b) Show that

$$\delta(ax) = \frac{1}{|a|} \delta(x)$$

4+2=6

UNIT—V

24. Solve the following equation using the method of separation of variables :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

where $v = 0$ for $y = 0$ and $y = a$ and $v = v_0$ for $x = -b$ and $x = b$. 6

25. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with the following boundary conditions : 6

$$u(0, t) = 0 \text{ and } u(l, t) = 0$$

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