FYUG Odd Semester Exam., 2023
(Held in 2024)

PHYSICS

(1st Semester)

Course No. : PHYDSC-101T

(Mathematical Physics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, taking *two* from each Unit :

2×10=20

UNIT—I

1. Which of the following obey commutative law?

$$\vec{A}+\vec{B},\quad \vec{A}-\vec{B},\quad \vec{A}\cdot\vec{B},\quad \vec{A}\times\vec{B}$$

2. Show that vector product of two vectors is a vector.

3. Define singular and non-singular matrices.

UNIT—II

4. Define order and degree of a differential equation.

5. What do you mean by ordinary differential equation (ODE)? Give an example of ODE.

6. When is a differential equation of the form $Mdx + Ndy = 0$ said to be exact or inexact?

UNIT—III

7. State and explain Stokes' theorem.

8. Define divergence of a vector. Whether divergence of a vector is scalar or vector?

9. Check and predict whether the vector $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ is irrotational vector or not.

UNIT—IV

10. What do you mean by orthogonal coordinate system?

11. Write down the transformation equations from Cartesian coordinates to cylindrical coordinates.

12. Write down the expressions of $\vec{\nabla}\phi$ and $\vec{\nabla} \cdot \vec{A}$ in spherical polar coordinates, where ϕ is a scalar function and $\vec{A}$ is a vector.

UNIT—V

13. What is meant by interpolation and extrapolation?

14. Write down Simpson's rule for integration of a function.

15. Evaluate $\Gamma(1)$.

SECTION—B

Answer *five* questions, taking *one* from each Unit : 10×5=50

UNIT—I

16\. (a) Prove that

$$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$$ 5

(b) Prove that

$$\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$$

where $\vec{a}$ is a vector. 5

17\. (a) Show that any square matrix can be uniquely expressed as the sum of symmetric matrix and antisymmetric matrix. 5

(b) Find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 3 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{bmatrix}$$ 5

UNIT—II

18\. (a) Solve the differential equation

$$(x^2+y^2)dx+2xy\,dy=0$$ 5

(b) Solve the differential equation

$$\frac{dy}{dx}+2xy=2e^{-x^2}$$ 5

19\. (a) What is auxiliary equation? If m_1 and m_2 are the two roots of the auxiliary equation, then write the expression of complementary function for the cases $m_1=m_2$ and $m_1\neq m_2$. 1+2=3

(b) Find the solution of the differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=x^2$$

when $y(0)=0$ and $y'(0)=\frac{1}{2}$. 7

UNIT—III

20\. (a) Show that $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=0$ and $\vec{\nabla}\times(\vec{\nabla}\phi)=0$. 3+3=6

(b) Show that $\vec{\nabla}\times(r^n\vec{r})=0$. 4

21. *(a)* State and prove Gauss' divergence theorem. 6

(b) Find the value of n for which the vector $r^n \vec{r}$ is solenoidal, where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$. 4

UNIT—IV

22. Find the expressions for line element and volume element in orthogonal curvilinear coordinates. Also find out the components of the vector $\vec{A} = 2y\hat{i} - 3\hat{j} + 2z\hat{k}$ in cylindrical polar coordinate system. 5+5=10

23. Deduce the expression for divergence, curl and Laplacian operator in general orthogonal coordinate system. 3+4+3=10

UNIT—V

24. *(a)* Define beta and gamma functions. Find the relation between them. 2+4=6

(b) Find the real root of the equation $x^3 - 9x + 1 = 0$. 4

25. *(a)* Explain bisection method of solving differential equation. 4

(b) Evaluate : 6

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}}$$

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