2023/TDC(CBCS)/ODD/SEM/
PHSHCC–101T/148

TDC (CBCS) Odd Semester Exam., 2023

PHYSICS

(Honours)

(1st Semester)

Course No. : PHSHCC–101T

(Mathematical Physics—I)

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. Explain transpose of a matrix with an example.

2. If A and B are non-singular matrices of same order, then show that

$$(AB)^{-1} = B^{-1}A^{-1}$$

3. Solve the differential equation :

$$9\frac{d^2y}{dx^2}-6\frac{dy}{dx}+y=0$$

Also find its Wronskian to show that its two solutions are independent.

UNIT—II

4. Prove that

$$\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B})=0$$

5. For vector $\vec{R}=x\hat{i}+y\hat{j}+z\hat{k}$, find the divergence.

6. Find a unit vector perpendicular to both of the vectors $\vec{A}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{B}=7\hat{i}-5\hat{j}+\hat{k}$.

UNIT—III

7. State Stokes' theorem.

8. Calculate the volume integral of $(\vec{\nabla}\cdot\vec{r})$ over the volume enclosed by a sphere of radius a, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

9. Find the directional derivative of

$$\phi=x^3+y^3+z^3$$

at the point (1, −1, 2) in the direction of the vector $\hat{i}+2\hat{j}+\hat{k}$.

UNIT—IV

10. Write the expression for Laplacian of a scalar in orthogonal curvilinear coordinates.

11. Write the expression for volume element in spherical polar coordinate system.

12. Write the expression for gradient of a scalar field in spherical polar coordinate system.

UNIT—V

13. Find the standard deviation of the following set of data :

4, 6, 8, 4, 10

14. What is meant by probability? Write the expression for probability function for binomial distribution.

15. What do you mean by independent random variables?

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. Find the eigenvalues and eigenvectors of the following matrix : 6

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

17. *(a)* Find the integrating factor and hence solve the differential equation

$$\frac{dy}{dx} + xy = 2x$$ 3

(b) Write the order and the degree of the differential equation

$$\frac{d^2y}{dx^2} + a^2 x = 0$$ 3

UNIT—II

18. *(a)* If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then show that $\vec{A}$ and $\vec{B}$ are mutually perpendicular. 3

(b) A particle moves from point (4, −3, −5) metre to point (−1, 4, 3) metre under the action of force $\vec{F} = (-3\hat{i} - \hat{j} + 2\hat{k})$ N. Find the work done by the force. 3

19. *(a)* Prove that for every field $\vec{v}$, div curl $\vec{v} = 0$. 3

(b) Calculate the curl of the vector

$$xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$$ 3

UNIT—III

20. State and prove Gauss's divergence theorem. 6

21. If $\vec{F} = \vec{\nabla}\phi$ everywhere in a region R and ϕ is single-valued and has continuous derivatives in R, then show that

$$\int_A^B \vec{F} \cdot d\vec{r}$$

is independent of the path joining the points A and B. 6

UNIT—IV

22. Find the gradient of $\phi = xyz$ in cylindrical coordinate system. Find the location of the positive roots of $x^3 - 9x + 1 = 0$, and evaluate the smallest one by bisection method correct to two decimal places. 3+3=6

23. Derive the expression for curl of a vector in orthogonal curvilinear coordinates. 6

UNIT—V

24. Show that mean and variance are equal in Poisson's distribution. 6

25. What is conditional probability? State and prove Bayes' theory. 2+4=6

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