TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(3rd Semester)

Course No. : MTMSEC-301T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer question from *either* Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MTMSEC-301T (A)

(Logic and Sets)

SECTION—A

Answer *fifteen* questions, taking any *three* from each Unit : 1×15=15

Unit—I

1. Write the negation of the following statement :

$$x \in \mathbb{N} \text{ and } 3x+7=0$$

2. If p is false, q is true, then what is the truth value of $p \vee \sim q$?

3. If 1 denotes a tautology, then show that $p \vee 1 \Leftrightarrow 1$.

4. Let $A = \{10, 20, 30, 40, 50, 60\}$. How many subsets of A have four elements?

Unit—II

5. Write the following statement using quantifier :

   "For all natural numbers, $n+3 > 2$."

6. If 0 denotes a contradiction, then show that

   $$p \wedge 0 \Rightarrow 0$$

7. Rewrite the following statement as an implication :

   "The diagonals of a parallelogram bisect each other."

8. Write the negation of each of the following :

   (a) There exists an infinite set whose power subsets are all finite.

   (b) For every real number x, there is an integer n such that $n > x$.

Unit—III

9. Justify True or False :

   $$A \subseteq B \Rightarrow A^c \subseteq B^c$$

10. What is $\mathbb{N} \cap (-3, 7)$?

11. If $A = \phi$, then write $P(P(A))$ in roster form.

12. How many subsets of B of $\{1, 2, 3, \cdots, n\}$ have the property that $B \cap \{1, 2\} = \phi$? Explain.

Unit—IV

13. Find the symmetric difference of the sets $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$.

14. Fill in the blank :

    $A \cap B = B$ iff _____.

15. Draw the Venn diagram of $(A \cap B^c \cap C)$ for any sets A, B and C

16. If $A = \phi$, then $|P(P(A))| = ?$

Unit—V

17. Define reflexive relation on a set.

18. Is every reflexive relation an identity relation? Justify.

19. Let $X = \{a, b, c, d\}$. Write a relation on X that is symmetric but neither transitive nor reflexive.

20. Define maximal element of a poset.

SECTION—B

Answer *five* questions, taking *one* from each Unit : 2×5=10

Unit—I

21. Construct the truth table for $(\sim p) \wedge (q \vee \sim p)$.

22. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Unit—II

23. What is the truth value of the quantification $(\exists x)$, $Q(x)$ if the statement $Q(x)$ and universe of discourse is given as follows?

$Q(x) : x^2 < 18$, $U =$ {positive integer not exceeding 4}

24. Show that there is no largest integer.

Unit—III

25. Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$ for any sets A, B, C.

26. Prove or disprove :

$$A \cap B = \phi \Rightarrow P(A) \cap P(B) = \phi$$

Unit—IV

27. Let $A_n = \{x \in \mathbf{Z} \mid x \geq n\}$ for each $n \in \mathbf{N}$. What is $\bigcap_{n=1}^{\infty} A_n$?

28. Find the value of k for which $A_3 \cap A_5 = A_k$, when $A_n = \{a \in \mathbf{Z} \mid a \leq n\}$.

Unit—V

29. Draw the Hasse diagram for the partial order $\leq$ on $\{0, 1, 2, 3\}$.

30. Show that the number of elements in the power set of a set having m elements is 2^m.

SECTION—C

Answer *five* questions, taking *one* from each Unit : 5×5=25

Unit—I

31. Using algebra of propositions, prove that $(s \Rightarrow ((\sim p \wedge q) \wedge r))$ is logically equivalent to $\sim ((p \vee (\sim (q \wedge r))) \wedge s)$.

32. *(a)* Prove that $p \to q = \sim p \vee q$. 2

(b) Show that an implication and its contrapositive are logically equivalent. 3

Unit—II

33. Using truth table, show that

$$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$$

is a tautology.

34. Using algebra of propositions, establish the following logical equivalences : 3+2

(i) $p \to (q \to r) \Leftrightarrow (p \wedge \sim r) \to \sim q$

(ii) $p \to (q \vee r) \Leftrightarrow (p \wedge \sim q) \to r$

Unit—III

35. Show that the union of two finite sets is finite.

36. *(a)* For any sets A, B, C, show that

$$(A \cap B) \times C = (A \times C) \cap (B \times C)$$ 3

(b) Show that $P(A \cap B) = P(A) \cap P(B)$. 2

Unit—IV

37. *(a)* Show that

$$\bigcap_{n=1}^{\infty} [0, \tfrac{1}{n}] = \{0\}$$ 3

(b) If $P(A) = P(B)$, then show that $A = B$. 2

38. *(a)* If A and B are nonempty sets, then show that $A \times B = B \times A$ iff $A = B$. 2

(b) Let $n \geq 1$ be a natural number. How many elements are in the set $\{(a, b) \in N \times \mathbb{N} \mid a \leq b \leq n\}$? Explain. 3

Unit—V

39. State and prove the fundamental theorem of equivalence relations.

40. Construct bijection from (1, 2) to (21, 2021). Justify your answer.

OPTION—B

Course No. : MTMSEC-301T (B)

(Programming in C)

SECTION—A

Answer *fifteen* questions, taking any *three* from each Unit : 1×15=15

Unit—I

1. Write the syntax to declare an integer variable.

2. How are comments written in a C program?

3. Mention two C library functions.

4. Write the syntax for scanf statement.

Unit—II

5. Write a C program to print the following line of text :

   Welcome to C

6. Convert the following equation to corresponding C statement :

$$A = \frac{(x+y)^2}{6} + xy^2$$

7. Identify the error, if any, in the following C statement :

   printf (x = % d, x);

8. What are the three logical operators in C?

Unit—III

9. Write the syntax of if-else statement.

10. What is the purpose of break statement?

11. If the initial value of x is 2, then what will be its value after execution of the statement $x++$; ?

12. Write the syntax of while loop.

Unit—IV

13. Write two advantages of using functions in C.

14. What are recursive functions?

15. What is the maximum number of arguments a function can have in C?

16. What should be the return type of a function that does not return any value?

Unit—V

17. Define an array.

18. What should be the type of an array index?

19. Point out errors, if any, in the following array declaration :

float $y(3)$;

20. Write the syntax to declare an integer array of size 10.

SECTION—B

Answer *five* questions, taking *one* from each Unit : 2×5=10

Unit—I

21. Write the rules for naming an identifier.

22. Distinguish between local and global variables.

Unit—II

23. Write a C program to input two integers and print their sum.

24. What will be the value of x when the following segment is executed?

int $x=10$, $y=15$;

$x=(x<y)?(y+x):y$;

Unit—III

25. Describe switch statement with a suitable example.

26. Write a C program to check whether a number (integer) is divisible by 2.

Unit—IV

27. Write a short note on function prototypes.

28. Distinguish between actual and formal arguments with regard to C functions.

Unit—V

29. What is the difference between an array and an ordinary variable? Explain with an example.

30. Write two advantages of using a multi-dimensional array in a C program.

SECTION—C

Answer *five* questions, taking *one* from each Unit : 5×5=25

Unit—I

31. Write a note on C data types.

32. Explain formatted input and output statements in C with suitable examples.

Unit—II

33. Write notes on logical and relational operators in C.

34. Write a C program to compute the area and perimeter of a rectangle, taking length and breadth as inputs.

Unit—III

35. Write a note on various conditional statements in C.

36. Discuss briefly about various loop control statements in C.

Unit—IV

37. Write a C program to find the factorial of a positive integer using recursion.

38. Discuss briefly about various types of functions in C, with suitable examples.

Unit—V

39. Discuss one-dimensional and multi-dimensional arrays.

40. Write a C program to find the least element in an array of integers.

OPTION—C

Course No. : MTMSEC-301T (C)

(Classical Algebra and Trigonometry)

SECTION—A

Answer *fifteen* questions, taking any *three* from each Unit : 1×15=15

Unit—I

1. Define idempotent matrix.
2. Define nilpotent matrix.
3. Define skew symmetric matrix.
4. Define unitary matrix.

Unit—II

5. Define normal form of a matrix.
6. Discuss consistency of a given system of equations.
7. What is the rank of zero matrix?
8. Define Echelon form of a matrix.

Unit—III

9. When is a polynomial equation said to be a reciprocal equation?
10. Write down the equation whose roots are $1, -2, 3, -4$.
11. Write down the sum of the roots of $x^3 - px^2 + qx - p = 0$.
12. Write the cubic equation, given roots are 1 and $1+i$.

Unit—IV

13. Write Gregory's series.
14. Find the value of $e^{i\pi/4}$.
15. State de Moiver's theorem.
16. If $x = \cos\theta + i\sin\theta$, then find x^2.

Unit—V

17. Expand $\cos(\alpha - i\beta)$ in the form of $a+ib$.
18. Define hyperbolic function.
19. Define $\sinh x$.
20. Write the relation between $\tan ix$ and $\tan hx$.

SECTION—B

Answer *five* questions, taking *one* from each Unit : 2×5=10

Unit—I

21. If A is a non-singular square matrix, then prove that $(A^1)^{-1} = (A^{-1})^1$.

22. Obtain the relation between rank of original matrix and its transpose.

Unit—II

23. Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$$

24. Define orthogonality of a square matrix.

Unit—III

25. Find the roots of $x^3 = 1$.

26. The sum of two roots of

$$x^3 + a_1x^2 + a_2x + a_3 = 0$$

is zero. Show that $a_1a_2 - a_3 = 0$.

Unit—IV

27. Find all possible values of $(-1)^{1/4}$.

28. If

$$\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$$

then prove that

(i) $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha + \beta + \gamma)$

(ii) $\sin3\alpha + \sin3\beta + \sin3\gamma = 3\sin(\alpha + \beta + \gamma)$

Unit—V

29. If n is positive integer, then prove that

$$(1+i)^n + (1-i)^n = 2^{\left(\frac{n}{2}+1\right)} \cos\frac{n\pi}{4}$$

30. Separate real and imaginary parts of $\tan(x + iy)$.

SECTION—C

Answer *five* questions, taking *one* from each Unit : 5×5=25

Unit—I

31. Find A^{-1} if the matrix

$$A = \begin{bmatrix} 4 & 3 & 8 \\ 6 & 2 & 5 \\ 1 & 5 & 9 \end{bmatrix}$$

32. If

$$A=\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$$

then show that adj(adj A) = A.

Unit—II

33. Solve :

$$x+2y+3z=11$$
$$x+2y+3z=-3$$
$$x+2y+3z=-1$$

34. If A and B are invertible square matrices of the same order, then prove that AB is invertible and $(AB)^{-1}=B^{-1}A^{-1}$.

Unit—III

35. If α, β, γ are roots of $x^3+px^2+qx+r=0$, then find the equation whose roots are $\beta+\gamma-2\alpha$, $\gamma+\alpha-2\beta$, $\alpha+\beta-2\gamma$.

36. *(a)* Solve $2x^3-x^2-18x+9=0$ if two roots are equal in magnitude but opposite on sign. 3

(b) Prove that $(x+1)^4=\alpha(x^4+1)$ is reciprocal if $\alpha \neq 1$. 2

Unit—IV

37. *(a)* If

$$x+\frac{1}{x}=2\cos\frac{\pi}{7}$$

then show that $x^7+\frac{1}{x^7}=-2$. 2

(b) Show that

$$\frac{\pi}{8}=\frac{1}{1\times3}+\frac{1}{5\times7}+\cdots$$ 3

38. If $x+iy=\sin(a+ib)$, then show that

$$\frac{x^2}{\cosh^2 b}+\frac{y^2}{\sinh^2 b}=1$$

Unit—V

39. Find the sum of

$$1-\frac{1}{3\cdot2^2}+\frac{1}{5\cdot2^4}-\cdots$$

40. Prove that

$$\pi=2\sqrt{3}\left[1-\frac{1}{3^2}+\frac{1}{5\times3^2}-\frac{1}{7\times3^3}+\cdots\right]$$

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