

**2023/TDC(CBCS)/ODD/SEM/
MTMHCC-303T/307**

TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(Honours)

(3rd Semester)

Course No. : MTMHCC-303T

(PDE and Systems of ODE)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

Answer ten questions, selecting any two from each

Unit : 2×10=20.

UNIT—I

- 1. Eliminate the arbitrary constants h and k from $(x-h)^2 + (y-k)^2 + \lambda^2 = z^2$ to form a PDE.**
- 2. Form a PDE from $ax^2 + by^2 + z^2 = 1$ by eliminating the arbitrary constants a and b .**
- 3. Construct a PDE from the function $u = f(x+ct) + g(x-ct)$ by eliminating f and g .**

(2)

UNIT—II

4. Obtain the canonical form of the PDE

$$2 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} + 8z = 0$$

5. Solve the PDE

$$2 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} = 1$$

using Lagrange's method.

6. Solve the PDE

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

by using separation of variables method.

UNIT—III

7. Find the characteristic of the PDE

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2y \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} - \frac{\partial z}{\partial x} - 6y = 0$$

8. Solve the PDE

$$(D^2 + 2DD' + D'^2)z = 0$$

9. Find the particular integral of the PDE

$$(D^2 - 2DD' + D'^2)z = \sin(2x + 3y)$$

(3)

UNIT—IV

10. Give interpretation of the variables
- x
- ,
- t
- and
- u
- in the heat conduction equation

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$$

11. Define homogeneous and non-homogeneous boundary conditions of a second-order PDE.

12. Find the eigenvalues of the differential equation

$$\frac{d^2 y}{dx^2} - \lambda^2 y = 0$$

UNIT—V

13. Define differential operator.

14. Solve :

$$\frac{dx}{dt} + \frac{1}{t}x = e^t$$

15. What is the matrix form of a system of non-homogeneous linear ordinary differential equations?

(4)

SECTION—B

Answer five questions, selecting one from each

Unit : 6×5=30

UNIT—I

16. (a) Form a PDE by eliminating the arbitrary function ϕ from $z = \phi(\sqrt{x^2 + y^2})$. 3

- (b) Construct a PDE from $x + y + z = f(x^2 + y^2 + z^2)$ by eliminating the arbitrary function f . 3

17. (a) Solve : 3

$$\frac{\partial z}{\partial y} + 2yz = y \sin x$$

- (b) Solve : 3

$$\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y} = 1$$

UNIT—II

18. (a) Solve the following PDE using Lagrange's method : 3

$$\frac{y^2 z}{x} \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = y^2$$

- (b) Solve the following PDE using method of separation of variables : 3

$$\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = (x + 2y)z$$

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(Continued)

(5)

19. Find the integral surface of the PDE

$$x(y^2 + z) \frac{\partial z}{\partial x} - y(x^2 + z) \frac{\partial z}{\partial y} = (x^2 - y^2)z$$

which contains the straight line $x + y = 0$,
 $z = 1$. 6

UNIT—III

20. (a) Solve the PDE 3

$$(D^2 + 3DD' + 2D'^2)z = x + y$$

- (b) Solve the PDE 3

$$(4D^2 + 4DD' + D'^2)z = 16 \log(x + 2y)$$

21. Find the canonical form of the PDE

$$\frac{\partial^2 z}{\partial x^2} + 5 \frac{\partial^2 z}{\partial x \partial y} + 6 \frac{\partial^2 z}{\partial y^2} = 0$$

and hence solve it. 6

UNIT—IV

22. Derive heat conduction equation in a bar. 6

23. Solve the PDE

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$$

subject to the conditions $u = 0$, $\frac{\partial u}{\partial x} = 1 + e^{-3y}$

when $x = 0$ for all y . 6

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(Turn Over)

(6)

UNIT—V

24. Solve the following system of ODEs : 6

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

25. Solve the following system of ODEs : 6

$$\frac{dx}{dt} + x = y + e^t$$

$$\frac{dy}{dt} + y = x + e^t$$

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