2023/FYUG/ODD/SEM/
MATSEC-101T/143

FYUG Odd Semester Exam., 2023
(Held in 2024)

MATHEMATICS

(1st Semester)

Course No. : MATSEC-101T

(Mathematical Skill Development with Software)

Full Marks : 50
Pass Marks : 20

Time : 2 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

Answer *fifteen* questions, selecting any *three* from
each Unit :   1×15=15

UNIT—I

1. What is an algorithm?

2. Draw the standard symbol to show any input/output operation in a flowchart.

(2)

3. What is meant by looping in programming?

4. What is meant by an array?

UNIT—II

5. How many functions can be defined from the set {1, 2, 3} to {1, 4, 2023}?

6. Draw the graph of the sine function on $\mathbb{R}$.

7. Draw the graph of the parabola

$$y = x^2 - 6x + 8$$

8. Define an odd function and give an example.

UNIT—III

9. State the division algorithm.

10. Define GCD of two integers.

11. What is the remainder, when 5^{2023} is divided by 24?

12. Find the number of positive divisors of 2023000.

(3)

UNIT—IV

13. What is an idempotent matrix?

14. If A is a 2023×2023 skew-symmetric matrix, then what is the determinant of A?

15. Find A^5 if

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

16. If A is a 10×10 matrix with determinant equal to 1, then what is the determinant of $2A$?

UNIT—V

17. What is the order and degree of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^3 + y = 0$$

18. Write the differential equation of the family of all parallel straight lines with slope 2.

19. Give example of a first-order homogeneous differential equation.

20. Write the integrating factor of the linear differential equation

$$\frac{dy}{dx} + Py = Q$$

(4)

SECTION—B

Answer *five* questions, selecting *one* from each Unit :   2×5=10

UNIT—I

21. Draw a diagram to explain decision making in flowcharts.

22. Draw a diagram to represent a pre-test iteration.

UNIT—II

23. Draw the graph of

$$f(x) = \begin{cases} x & \text{if } x \geq 1 \\ x^2 & \text{if } x < 1 \end{cases}$$

24. If f is an odd function, what can you conclude about $f(0)$? Justify your answer.

UNIT—III

25. Find the GCD of 12321 and 8658 using Euclidean algorithm.

26. Show that the integer $53^{103} + 103^{53}$ is divisible by 39.

(5)

UNIT—IV

27. Find the adjoint of the following matrix :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 3 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

28. Express

$$A = \begin{pmatrix} 1 & 7 & 9 \\ 2 & -1 & 3 \\ 0 & 1 & 3 \end{pmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix.

UNIT—V

29. Find the differential equation of the family of curves

$$y = A\cos(\omega x + B)$$

where A and B are arbitrary constants, ω is fixed.

30. Solve the differential equation :

$$\cos y\, dx + (1 + e^x)\sin y\, dy = 0$$

SECTION—C

Answer *five* questions, selecting *one* from each Unit : 5×5=25

UNIT—I

31. *(a)* Write an algorithm to check if a number is even or odd. 2

(b) Draw a flowchart to find the sum of first n natural numbers. 3

32. *(a)* Write an algorithm to illustrate the interchange of the values of two variables. 2

(b) Draw a flowchart to construct an array of a desired size. 3

UNIT—II

33. *(a)* Construct a bijection from $\mathbb{N}$ to $\mathbb{N}$, other than the identity function. Here $\mathbb{N}$ is the set of natural numbers. Justify your answer. 3

(b) If $f(x) = x^2 + 2x + 3$ and $g(x) = \sin 2x$ for $x \in \mathbb{R}$, define $f \circ g$ and $g \circ f$. 2

34. *(a)* Give example of a one-one function that is not onto. Also give example of an onto function that is not one-one. Justify your answer. 4

(b) If $1 < x < 2$, then what is the value of

$$[x] + [x+1] + \left[\frac{1}{x}\right]$$

where $[\cdot]$ is the greatest integer function? 1

UNIT—III

35. *(a)* Prove that there are infinitely many primes. 3

(b) If $a \equiv b \pmod n$ and $b \equiv c \pmod n$, then show that $a \equiv c \pmod n$. 2

36. *(a)* Let a and b be integers, not both zero. Show that a and b are relatively prime if and only if there exists integers x and y such that $ax + by = 1$. 3

(b) Find integers x and y such that

$$306x + 657y = \gcd(306, 657)$$ 2

(8)

UNIT—IV

37. State and prove Jacobi's theorem.   1+4=5

38. Find inverse of the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 5 & 1 & -1 \\ 2 & -3 & -3 \end{pmatrix}$$

and verify that $AA^{-1} = I.$ 5

UNIT—V

39. Solve :   5

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

40. Solve :   5

$$\frac{dy}{dx} + 2xy = xy^3$$

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