2023/FYUG/ODD/SEM/
MATDSC–101T/140

FYUG Odd Semester Exam., 2023

(Held in 2024)

MATHEMATICS

(1st Semester)

Course No. : MATDSC–101T

(Higher Algebra and Trigonometry)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, taking *two* from each Unit :

2×10=20

UNIT—I

1. Find the general value of θ which satisfies the following equation :

$$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\cdots(\cos n\theta + i\sin n\theta) = 1$$

2. Find all the values of $(-1)^{1/3}$.

3. Expand $\sin^3 x$ in ascending powers of x.

UNIT—II

4. Prove that $i^i = e^{-(4n+1)\pi/2}$.

5. Show that

$$\pi = 2\sqrt{3}\left\{1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right\}$$

6. Express $\sin(x+iy)$ in the form of $A+iB$.

UNIT—III

7. The relation R on $\mathbb{Z}$ defined by $(a, b) \in R$ iff $|a-b| = 2023$. Show that R is symmetric but not transitive.

8. Let R be an equivalence relation on A. Show that for any $a, b \in A$, $[a] = [b]$ iff $a \in [b]$, where the symbol [ ] represents equivalence class.

9. Write the negation of the statement $\forall \alpha \in A, \exists x \in B$ such that $x > \alpha$.

UNIT—IV

10. If α, β and γ are the roots of the equation $x^3 - 3x^2 + 2x - 7 = 0$, then find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.

11. Find the equation whose roots are reciprocal of the roots of $x^3 - 6x^2 + 11x - 6 = 0$.

12. If $x+y+z=1$, then prove that

$$(1-x)(1-y)(1-z) > 8xyz$$

UNIT—V

13. What do you mean by canonical form of matrices?

14. Define rank of a matrix.

15. Show that the set $\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ is LI.

SECTION—B

Answer *five* questions, taking *one* from each Unit : 10×5=50

UNIT—I

16. *(a)* State de Moivre's theorem and prove it for positive integral index. 4

(b) (i) If $(1+x)^n = a_0 + a_1x + a_2x^2 + \cdots$, then prove that

$$a_0 - a_2 + a_4 - \cdots = 2^{n/2}\cos^{n\pi/4}$$ 3

(ii) Expand $\cos 7\theta$ in ascending powers of $\cos\theta$. 3

17. (a) (i) If $x = \cos\theta + i\sin\theta$ and $1+\sqrt{1-a^2} = na$, then prove that

$$1 + a\cos\theta = \frac{a}{2n}(1+nx)\left(1+\frac{n}{x}\right)$$ 3

(ii) Expand $\sin x$ in ascending powers of x. 3

(b) Prove that

$$\frac{\sin^3\theta}{\lfloor 3} = \frac{\theta^3}{\lfloor 3} - (1+3^2)\frac{\theta^5}{\lfloor 5} + (1+3^2+3^4)\frac{\theta^7}{\lfloor 7} - \cdots$$ 4

UNIT—II

18. (a) State and prove Gregory's series. 4

(b) (i) If $\cos^{-1}(\alpha + i\beta) = x + iy$, then show that $\alpha^2 \operatorname{sech}^2 y + \beta^2 \operatorname{cosech}^2 y = 1$. 3

(ii) Find the sum of the series

$$\cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta - \frac{1}{4}\cos 4\theta + \cdots$$ 3

19. (a) (i) Prove that

$$\log(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{y}{x}$$ 3

(ii) If θ lies between 0 and $\pi/2$, then prove that

$$\tan^{-1}\left(\frac{1-\cos\theta}{1+\cos\theta}\right) = \tan^2\frac{\theta}{2} - \frac{1}{3}\tan^6\frac{\theta}{2} + \frac{1}{5}\tan^{10}\frac{\theta}{2} - \cdots$$ 3

(b) Find the sum of the series

$$\cos\theta + \frac{\operatorname{cosec}\theta}{\lfloor 1}\cos 2\theta + \frac{\operatorname{cosec}^2\theta}{\lfloor 2}\cos 3\theta + \cdots$$ 4

UNIT—III

20. (a) State that the relation of 'congruence modulo n' is an equivalence relation on $\mathbb{Z}$. 5

(b) Show that—

(i) $(p \wedge q) \Rightarrow (p \vee q)$ is a tautology;

(ii) $(\sim p \wedge q) \wedge (p \vee (\sim q))$ is a contradiction. 2+3=5

21. (a) Show that a partition of a non-empty set induces an equivalence relation on A such that the equivalence classes are precisely the members of A. 5

(b) (i) Write the following statement using quantifiers and other symbols as required :

For every positive real number ε, there exists a natural number n_0 such that the reciprocal of n_0 is less than ε. 2

(ii) Write the following statement as an implication :

If x is greater than 2, then x^2 is greater than 4.

Also, write its converse and contra-positive. 3

UNIT—IV

22. (a) (i) The sum of two roots of the equation $x^3 + a_1x^2 + a_2x + a_3 = 0$ is zero. Show that $a_1a_2 = a_3$. 3

(ii) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$. 3

(b) State and prove Cauchy-Schwarz inequality. 4

23. (a) (i) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2\beta$. 3

(ii) Find the nature of the roots of the equation $x^3 + x^2 - 16x + 20 = 0$. 3

(b) Solve $x^3 - 30x + 133 = 0$ by Cardan's method. 4

UNIT—V

24. (a) Show that the rank of the transpose of a matrix is the same as that of the original matrix. 5

(b) Solve by Gaussian elimination method : 5

$$x + y + z = 6$$
$$2x - y + 2z = 6$$
$$2x + 2y + z = 9$$

25. (a) Find the rank of the matrix

$$\begin{pmatrix} 2 & 2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ -1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$$

by reducing it to normal form. 5

(b) *(i)* Prove that every singleton set containing non-zero vector is LI. 2

(ii) Show that the vectors (1, 1, 0), (1, 3, 5) and (2, 2, 0) in $\mathbb{R}^3$ are LD. 3

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