2023/TDC(CBCS)/ODD/SEM/
MTMHCC–102T/303

TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(Honours)

(1st Semester)

Course No. : MTMHCC–102T

(Higher Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. Find all the values of $1^{1/3}$.

2. Expand $\cos^2 \theta$ in powers of θ.

3. Show that i^i is purely real.

UNIT—II

4. Define equivalence relation.

5. Give an example of a relation which is reflexive and transitive, but not symmetric.

6. If $f : A \to B$ and $g : B \to C$ are both one-one, show that $g \circ f$ is one-one.

UNIT—III

7. State the principle of mathematical induction.

8. State the well-ordering principle of N.

9. State division algorithm.

UNIT—IV

10. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, find $\Sigma\alpha$.

11. State Descarte's rule of signs.

12. How many possible positive roots can the following equation have?

$$x^4 + 2x^3 + 3x - 1 = 0$$

UNIT—V

13. Define echelon form of a matrix.

14. Define row canonical form of a matrix.

15. Define rank of a matrix.

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 10×5=50

UNIT—I

16. State and prove de Moivre's theorem for rational indices.

17. *(a)* If $z_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, then prove that $z_1 \, z_2 \, z_3 \cdots = -1$. 5

(b) If

$$x = \frac{2}{1!} - \frac{4}{3!} + \frac{6}{5!} - \frac{8}{7!} + \cdots,$$

$$y = 1 + \frac{2}{1!} - \frac{2^3}{3!} + \frac{2^5}{5!} - \cdots,$$

then show that $x^2 = y$. 5

UNIT—II

18. Show that the relation R on $\mathbb{Z}$ defined by $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a - b$ is divisible by $7\}$ is an equivalence relation. What are the distinct equivalence classes in $\mathbb{Z}$ under this relation?

19. If $f : A \to B$ and $g : B \to C$ are two functions such that $g \circ f$ is one-one and onto, show that f is one-one and g is onto. Give examples to establish that f need not be onto and g need not be one-one.

UNIT—III

20. Prove that no integer in the following sequences is a perfect square :

(a) 11, 111, 1111, 11111, ⋯

(b) 99, 999, 9999, 99999, ⋯

21. Let $n \in \mathbb{N}$ and $a, b \in \mathbb{Z}$. Show that—

(a) if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$;

(b) if $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$ for any $k \in \mathbb{N}$.

UNIT—IV

22. Solve by Cardan's method

$$x^3 + 9x^2 + 15x - 25 = 0$$

23. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, find—

(i) $\Sigma\alpha^2$

(ii) $\Sigma\alpha^3$

(iii) $\Sigma\frac{1}{\alpha}$

(iv) $\Sigma\alpha^2\beta$

UNIT—V

24. Reduce the following matrix to row canonical form and hence find its rank :

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

25. Solve the following system of linear equations by Gaussian backward elimination method :

$$
\begin{aligned}
x_1 + 2x_2 - x_3 + 4x_4 &= 5 \\
2x_1 - x_2 + 3x_3 + x_4 &= 3 \\
x_1 + 4x_2 + 3x_3 - 7x_4 &= 1 \\
3x_1 - 6x_2 + 4x_3 - 11x_4 &= -2
\end{aligned}
$$

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