TDC (CBCS) Odd Semester Exam., 2023

MATHEMATICS

(Honours)

(1st Semester)

Course No. : MTMHCC–101T

(Calculus)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer *ten* questions, selecting any *two* from each Unit : 2×10=20

UNIT—I

1. If $y = \log x$, then find y_n.

2. If $y = c_1 \cos(\log x) + c_2 \sin(\log x)$, then show that

$$x^2 y_2 + xy_1 + y = 0$$

3. Using Leibnitz's theorem, differentiate n times the equation

$$(1+x^2)y_2+(2x-1)y_1=0$$

UNIT—II

4. Define asymptote of a curve.

5. Evaluate $\lim_{x\to 0}\frac{\sec x-1}{x\sec x}$.

6. Find the points of inflexion on the curve $y=(\log x)^3$.

UNIT—III

7. If $I_n=\int x^n\cos ax\,dx$ and $J_n=\int x^n\sin ax\,dx$, then show that

$$aI_n=x^n\sin ax-nJ_{n-1}$$

8. If $I_n=\int_0^{\pi/2}\sin^n x\,dx$, n being a +ve integer, then show that

$$I_n=\frac{n-1}{n}I_{n-2}$$

9. Evaluate $\int_0^{\pi/2}\cos^{10}x\,dx$.

UNIT—IV

10. Find by integration the length of the line $y=5x$ from $x=0$ to $x=5$.

11. Find the area bounded by the parabola $y^2=4ax$ and its latus rectum.

12. Find the surface area of a sphere of radius r.

UNIT—V

13. Show that $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$ and $\vec{c}\times(\vec{a}\times\vec{b})$ are coplanar.

14. Find the vector equation of a sphere whose centre is $2\hat{i}-\hat{j}+\hat{k}$ and radius 5 units.

15. Show that the derivative of a constant vector is zero.

SECTION—B

Answer *five* questions, selecting *one* from each Unit : 6×5=30

UNIT—I

16. (a) If $y=e^{a\sin^{-1}x}$, then prove that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$$

Hence find $(y_n)_0$. 3

(b) If $y = x^{2n}$, where n is a +ve integer, then show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5 \cdot \ldots (2n-1)\} x^n$$ 3

17. (a) If $ax^2 + 2hxy + by^2 = 1$, then show that

$$y_2 = \frac{h^2 - ab}{(hx+by)^3}$$ 3

(b) If $y = x^{n-1} \log x$, then show that

$$y_n = \frac{(n-1)!}{x}$$ 3

UNIT—II

18. (a) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ be finite, then find the value of a and the limit. 3

(b) Trace the curve $r = a(1 + \cos\theta)$. 3

19. (a) Obtain the asymptotes of the curve $x^3 + y^3 - 6x^2 = 0$. 3

(b) Examine the concavity of the curve $y = x^3 - 3x + 3$ and find the points of inflexion, if any. 3

UNIT—III

20. (a) If $I_{m,n} = \int \sin^m x \cos^n x \, dx$, then prove that

$$I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \left(\frac{n-1}{m+n}\right) I_{m,n-2}$$ 3

(b) Obtain a reduction formula for $I_{m,n} = \int \cos^m x \cos nx \, dx$ connecting $I_{m-1,n-1}$. 3

21. (a) If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then show that

$$n(I_{n-1} + I_{n+1}) = 1$$ 3

(b) If $I_n = \int_0^{\pi/2} x^n \sin x \, dx$, $n > 0$, then show that

$$I_n + n(n-1) I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$ 3

UNIT—IV

22. (a) Find the area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$. 3

(b) Show that the perimeter of the curve $r = a(1 - \cos\theta)$ is $8a$. 3

23. *(a)* Find the area of the asteroid $x^{2/3}+y^{2/3}=a^{2/3}$. 3

(b) Find the surface area of the solid generated by revolving the cycloid $x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ about its base. 3

UNIT—V

24. *(a)* Show that if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar, then $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also non-coplanar. Is this true for $\vec{a}-\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$? 2+1=3

(b) Find the vector equation of a plane passing through two given points and parallel to a given vector. 3

25. *(a)* Prove that the necessary and sufficient condition for $\vec{r}=\vec{f}(t)$ to have a constant direction is $\vec{f}\times\dfrac{d\vec{f}}{dt}=\vec{0}$. 3

(b) If $\vec{r}=3t\hat{i}+3t^2\hat{j}+2t^3\hat{k}$, where t is a scalar, then show that

$$\left[\frac{d\vec{r}}{dt}\;\frac{d^2\vec{r}}{dt^2}\;\frac{d^3\vec{r}}{dt^3}\right]=216$$ 3

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