2021/TDC (CBCS)/EVEN/SEM/ MTMDSE–602T/129

TDC (CBCS) Even Semester Exam., September—2021

MATHEMATICS

(6th Semester)

Course No. : MTMDSE–602T

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer *either* from Option—A *or* from Option—B

OPTION—A

Course No. : MTMDSE–602T (A)

(Hydrodynamics)

SECTION—A

Answer any *twenty* of the following as directed : 1×20=20

1. What do you mean by ideal fluids?

2. Give three examples of real fluid.

3. What is viscosity?

4. What do you mean by laminar flow?

5. Define steady flow.

6. What do you mean by uniform flow?

7. Define barotropic flow.

8. Name the two forces that act on a fluid mass.

9. What are the two methods of describing fluid motion?

10. When do streamlines and path lines coincide?

11. Does $\vec{q} \times d\vec{r} = \vec{0}$ represent path lines? If not, what does it represent?

12. If $\vec{q} = -\vec{\nabla}\phi$, then ϕ is called ____.

(Fill in the blank)

13. If curl $\vec{q} = \vec{0}$, what can you say about the flow?

14. Velocity potential exists only for ____ motion.

(Fill in the blank)

15. Write the physical significance of equation of continuity.

16. What is the equation of continuity for homogeneous incompressible fluid? (Vector form)

17. The elementary mass in spherical polar coordinates is

(a) $\rho r \sin^2\theta \, dr \, d\theta \, d\phi$

(b) $\rho r \sin^2\theta \, dr \, d\theta \, d\phi$

(c) $\rho r^2 \sin\theta \, dr \, d\theta \, d\phi$

(d) None of the above

(Choose the correct answer)

18. If $u = -lx$, $v = my$, $w = (l - m)z$ are velocity components of an incompressible fluid, is the motion possible? (l, m, n are constants)

19. Write the equation of continuity in Lagrangian form.

20. $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho_r q_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho q_\theta) + \frac{\partial}{\partial z}(\rho q_z) = 0$ is the equation of continuity in ____ coordinates.

(Fill in the blank)

21. Write the equation of continuity for an incompressible fluid in Cartesian coordinates.

22. Stream function of flow satisfies

(a) Euler's equation

(b) Lagrange's equation

(c) Laplace's equation

(d) None of the above

(Choose the correct answer)

23. Acceleration of a fluid particle is the ____ derivative of fluid velocity.

(Fill in the blank)

24. In usual notations, acceleration of a fluid particle is $\frac{\partial \vec{q}}{\partial t}$.

(State True or False)

25. $\frac{D}{Dt}$ is known as

(a) static differential operator

(b) partial differential operator

(c) total differential operator

(d) differentiation following the motion

(Choose the correct answer)

26. If in two-dimensional motion, fluid velocity is given by

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

then what is the name of the function ψ?

27. Write the relation among local, material and convective derivatives.

28. Write Euler's equation of motion in X-direction.

29. What is the energy equation for incompressible fluid?

30. What do you mean by conservative force?

31. Euler's equation of motion is a statement of

(a) linear momentum conservation for the flow of an inviscid fluid

(b) mass conservation

(c) energy conservation

(d) None of the above

(Choose the correct answer)

32. Euler's equation of motion for a steady flow of an ideal fluid along a streamline is based on Newton's ____ of motion.

(Fill in the blank)

33. Euler's equation is suitable for which of the following cases?

(a) Compressible flow

(b) Incompressible flow

(c) Both *(a)* and *(b)*

(d) None of the above

(Choose the correct answer)

34. Write the equation of motion of a homogeneous inviscid liquid moving under conservative force.

35. Write Lamb's hydrodynamical equation.

36. Bernoulli's equation is obtained by ____ Euler's equation of motion.

(Fill in the blank)

37. If the motion is steady, velocity potential does not exist and V be the potential function from which the external forces are derivable, then Bernoulli's theorem is

(a) $-\frac{\partial\phi}{\partial t}+\frac{1}{2}q^2+V\int\frac{dp}{d\rho}=C$

(b) $-\frac{\partial\phi}{\partial t}+\frac{1}{2}q^2+V+\frac{p}{\rho}=C$

(c) $\frac{p}{\rho}+\frac{q^2}{2}+V=C$

(d) None of the above

(Choose the correct answer)

38. Bernoulli's equation for unsteady and irrotational motion is given by

(a) $-\frac{\partial \phi}{\partial t}+\frac{q^2}{2}+V+\frac{p}{\rho}=F(t)$

(b) $-\frac{\partial \phi}{\partial t}+\frac{q^2}{2}+V=F(t)$

(c) $-\frac{\partial \phi}{\partial t}-\frac{q^2}{2}+V-\frac{p}{\rho}=F(t)$

(d) $\frac{q^2}{2}+V+\frac{p}{\rho}=F(t)$

(Choose the correct answer)

39. According to Euler's momentum theorem, net rate of gain of momentum is

(a) $\rho(\alpha_2^2 q_2-\alpha_1^2 q_1)$

(b) $\rho(\alpha_2 q_2^2-\alpha_1 q_1^2)$

(c) $\alpha_2 q_2^2-\alpha_1 q_1^2$

(d) None of the above

(Choose the correct answer)

40. State D'Alembert's paradox.

SECTION—B

Answer any *five* of the following questions : 2×5=10

41. Write the difference between streamlines and path lines.

42. If the components of fluid velocity are given by

$$u = kyz, \quad v = kzx, \quad w = kxy$$

then show that the flow is irrotational.

43. Test whether the motion specified by $u = Cx$, $v = Cy$, $w = -2Cz$ is a possible motion for an incompressible fluid. (C is constant)

44. Show that the equation of continuity reduces to Laplace's equation when the liquid is incompressible and irrotational.

45. Write the components of acceleration of a fluid particle in Cartesian coordinates.

46. Define stream function.

47. Obtain Cartesian form of Euler's equation of motion from vector form.

48. Write the statement of energy equation.

49. State Bernoulli's theorem.

50. Explain Euler's momentum theorem.

SECTION—C

Answer any *five* of the following questions : 8×5=40

51. Explain Lagrangian and Eulerian methods of describing fluid motion. 4+4=8

52. Find the streamlines and path lines of the particles when

$$u=\frac{x}{1+t},\quad v=\frac{y}{1+t},\quad w=\frac{z}{1+t}$$ 4+4=8

53. Derive the equation of continuity in the form

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{q})=0$$ 8

54. A mass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinders. Show that the equation of continuity is

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho_u)+\frac{\partial}{\partial z}(\rho_v)=0$$ 8

55. Establish the result

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (\vec{q} \cdot \vec{\nabla})$$

What are material, local and convective derivatives? 5+3=8

56. *(a)* Show that stream function satisfies Laplace's equation.

(b) Given the velocity field

$$\vec{q} = (Ax^2y)\hat{i} + (By^2zt)\hat{j} + (Czt^2)\hat{k}$$

Determine the acceleration of a fluid particle of fixed identity. 3+5=8

57. Establish Euler's equation of motion for an inviscid fluid in the form

$$\frac{d\vec{q}}{dt} - \vec{F} + \frac{1}{\rho}\vec{\nabla}p = 0$$ 8

58. Prove that the equation of motion of a homogeneous inviscid liquid moving under conservative forces may be written in the form

$$\frac{\partial\vec{q}}{\partial t} - \vec{q} \times \text{curl}\,\vec{q} = -\vec{\nabla}\left(\Omega + \frac{p}{\rho} + \frac{1}{2}q^2\right)$$ 8

59. Prove that when velocity potential exists and forces are conservative and derivable from a potential Ω, the equation of motion can always be integrated and the solution is

$$\int \frac{dp}{\rho} - \frac{\partial \phi}{\partial t} + \frac{1}{2} q^2 + \Omega = F(t)$$ 8

60. A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A, is delivered at atmospheric pressure at a place where the sectional area is B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth

$$\frac{S^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$$

below the pipe, S being the delivery per second. 8

OPTION—B

Course No. : MTMDSE–602T (B)

(Theory of Equation)

SECTION—A

Answer any *twenty* of the following questions : 1×20=20

1. Write the general form of a polynomial of degree n with real coefficients.

2. If $P(x)$ is a polynomial of degree 2, what is the geometrical figure represented by the graph of $y = P(x)$?

3. What is the maximum value of $P(x) = -x^2 + 4x + 7$?

4. What is the minimum value of $P(x) = x^2 - x + 3$?

5. If α is a zero of a polynomial $Q(x)$, then what is $Q(\alpha)$?

6. State Descartes rule of signs.

7. If $P(x)$ and $Q(x)$ are polynomials of degrees m and n respectively, then what is the degree of the polynomial $P(x) + Q(x)$?

8. What can you conclude about the roots of $x^4+x^3+x^2+1=0$ using Descartes rule?

9. Find the sum of roots of the equation

$$2x^5-7x^4+x^2-x+1=0$$

10. If α and β are the roots of $x^2-3x+2=0$, then find $\frac{1}{\alpha}+\frac{1}{\beta}$.

11. If α, β, γ are the roots of a cubic equation, then express $\sum\alpha^2$ in terms of $\sum\alpha$ and $\sum\alpha\beta$.

12. What is the equation whose roots are reciprocals of the roots of $x^3+4x^2+x+1=0$?

13. If one root of $x^3+4x^2+ax+b=0$ is negative of the other, then find the third root.

14. Form the equation whose roots are opposite in signs to those of $x^5-x^2+x-3=0$.

15. All roots of $x^3+ax^2+b=0$ are equal to 4, what is the value of a?

16. Justify True or False :

All roots of the equation $x^5+x^3+7x-3=0$ are imaginary.

17. Name a general method to solve a cubic equation.

18. How many roots does a biquadratic equation have?

19. What are the zeros of the cubic $P(x) = (x+1)(x^2 - 4x + 4)$?

20. What are the roots of the biquadratic equation $(x^2 - 3x + 2)(x^2 + x - 2) = 0$?

21. Justify True or False :

If two roots of a biquadratic equation are $1+2i$ and $2+i$, then the other two roots are not real.

22. If α, β, γ are the roots of a cubic equation, then express $\sum \alpha^2\beta$ in terms of $\sum \alpha$, $\sum \alpha\beta$ and $\sum \alpha\beta\gamma$.

23. If one root of a cubic equation is $2-3i$, write at least one other root.

24. What are the roots of the cubic $x^3 - 8 = 0$?

25. Define the sum of the homogeneous products of r dimensions of n quantities $a_1, a_2, \cdots, a_n$.

26. Define superior limit of the positive roots of an equation.

27. Define inferior limit of the positive roots of an equation.

28. State Sturm's theorem.

29. State Strum's theorem for the case of equal roots.

30. State the condition for the reality of all the roots of any equation.

31. Write the standard form of a biquadratic in terms of G, H and I.

32. Define limiting equations.

33. State the theorem of Fourier and Budan.

34. Write De Gua's rule for finding imaginary roots.

35. Write the conditions for the reality of the roots of a biquadratic.

36. Name two methods of solution of numerical equations.

37. What are commensurable and incommensurable classes of roots?

38. Justify True or False :

An equation in which the coefficient of the first term is unity and the coefficients of the other terms are whole numbers cannot have a commensurable root which is not a whole number.

39. Check if the roots of the equation $x^3 - 2x = 0$ are commensurable or incommensurable.

40. Find an interval containing a root of the equation $x^3 - 2x^2 - 3x + 2 = 0$.

SECTION—B

Answer any *five* of the following questions : 2×5=10

41. Plot the graph of $y = x^2 + 6x + 10$.

42. Find the quotient and remainder when $x^3 + 5x^2 + 3x + 2$ is divided by $x - 1$.

43. Find the cubic equation, two of whose roots being 1 and $3 + 2i$.

44. Let α, β, γ be the roots of $x^3 + px^2 + qx + r = 0$. Find the value of $\sum \alpha^2$.

45. If α, β, γ are roots of $x^3 + qx + r = 0$, then find the equation whose roots are

$$\frac{\beta+\gamma}{\alpha^2}, \frac{\gamma+\alpha}{\beta^2}, \frac{\alpha+\beta}{\gamma^2}$$

46. Using derived function, find the maximum/minimum values of

$$f(x) = 2x^3 - 3x^2 - 36x + 14$$

47. Find a superior limit of the positive roots of the equation

$$x^4 - 5x^3 + 40x^2 - 8x + 23 = 0$$

48. Find the nature of roots of the equation

$$x^4 - 5x^3 + 9x^2 - 7x + 2 = 0$$

49. Find the conditions that the roots of the cubic $z^3 + 3Hz + G = 0$ should be all real and unequal.

50. Find an interval containing a root of

$$x^3 - 2x - 5 = 0$$

and compute the first two approximations of that root by Newton's method.

SECTION—C

Answer any *five* of the following questions : 8×5=40

51. *(a)* Show that the equation $x^4+x^2+x-2=0$ has two real and two imaginary roots. 3

(b) Assuming the fundamental theorem of algebra, show that a polynomial of degree n has exactly n zeros. 5

52. *(a)* Given that the equation

$$x^4-x^3-7x^2+x+6=0$$

has one of its roots as −2, find the other roots. 4

(b) Solve the equation

$$x^4-10x^3+29x^2-22x+4=0$$

if one of the roots is $2+\sqrt{3}$. 4

53. *(a)* If α, β, γ are the roots of the equation

$$x^3+px^2+qx+r=0$$

then form the equation whose roots are $\beta+\gamma-2\alpha$, $\gamma+\alpha-2\beta$ and $\alpha+\beta-2\gamma$. Hence find the value of

$$(\beta+\gamma-2\alpha)(\gamma+\alpha-2\beta)(\alpha+\beta-2\gamma)$$ 4

(b) If the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in harmonic progression, then prove that $2q^3 = r(3pq - r)$. 4

54. (a) Solve $x^3 - 7x + 36 = 0$, given that one root is double the other root. 4

(b) Find the sum of the fourth powers of the roots of the equation

$$x^3 - 2x^2 + x - 1 = 0$$ 4

55. (a) Solve by Cardan's method : 4

$$x^3 - 6x - 4 = 0$$

(b) Form the equation whose roots are the several values of $\frac{\alpha + \beta}{2}$ where α, β, γ, δ are the roots of a biquadratic. 4

56. (a) If α, β, γ, δ are the roots of the equation

$$a_0 x^4 + 4a_1 x^3 + 6a_2 x^2 + 4a_3 x + a_4 = 0$$

then solve the equation

$$\sqrt{x - \alpha} + \sqrt{x - \beta} + \sqrt{x - \gamma} + \sqrt{x - \delta} = 0$$

in terms of the coefficients a_0, a_1. 4

(b) Solve the cubic equation

$$x^3 - 3x^2 + 12x + 16 = 0$$ 4

57. *(a)* Discuss the nature of roots of the equation $x^4 + 4x^3 - 2x^2 - 12x + 5 = 0$. 4

(b) Analyze the situation of roots of the equation $x^5 + x^4 - 4x^3 - 3x^2 + 3x + 1 = 0$. 4

58. *(a)* Find the nature of roots of the equation

$$x^5 + 2x^4 + x^3 - x^2 - 2x - 1 = 0$$ 3

(b) Analyze the equation

$$2x^6 - 18x^5 + 60x^4 - 120x^3 - 30x^2 + 18x - 5 = 0$$ 5

59. *(a)* Write a short note on Newton-Raphson method. 4

(b) Find the integer roots of the equation

$$x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$$ 4

60. *(a)* Find the roots of

$$x^5 - 23x^4 + 160x^3 - 281x^2 - 257x - 440 = 0$$

by the method of divisors. 4

(b) Find an approximate positive root of

$$x^3 - 6x - 13 = 0$$

using Newton's method. 4

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